Multispacecraft measurement of fundamental turbulence properties: progress and outstanding problems

Lectures of an introductory nature

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The heliospheric space plasma physics in the era of multipoint space missions
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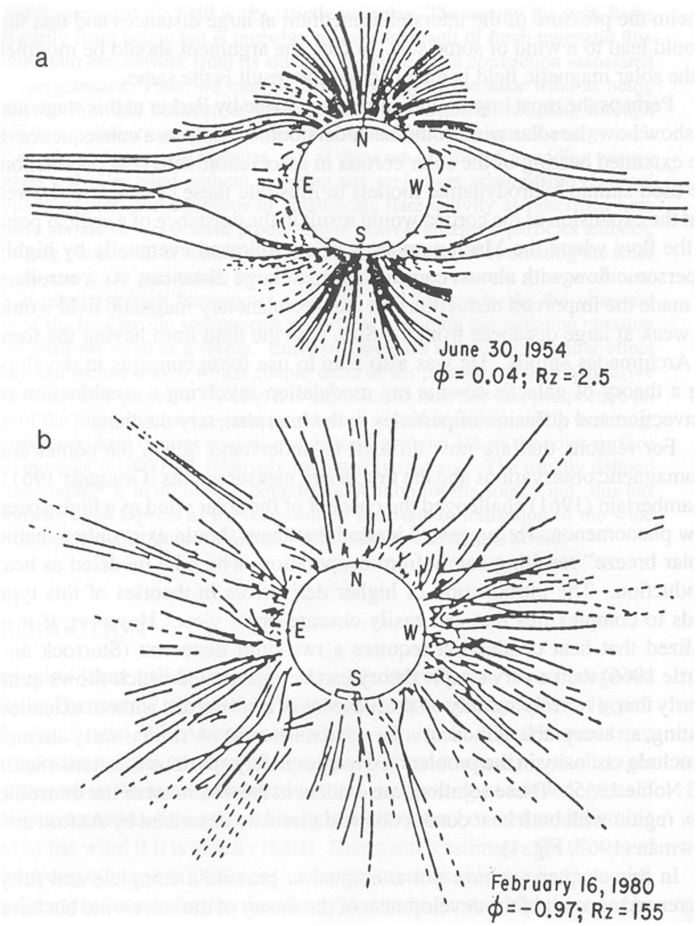
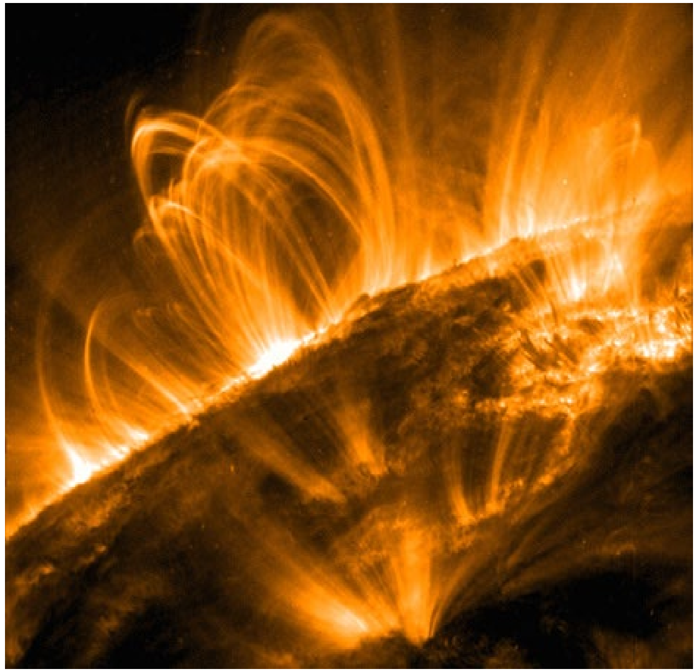
I. Context

Fine scale activity in the corona

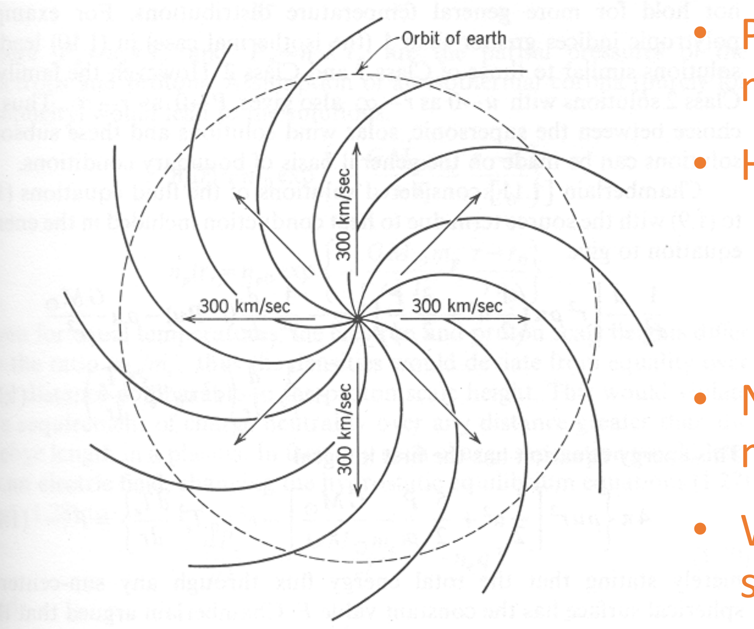


Drawings from Coronagraphs
(Loucif and Koutchmy, 1989)

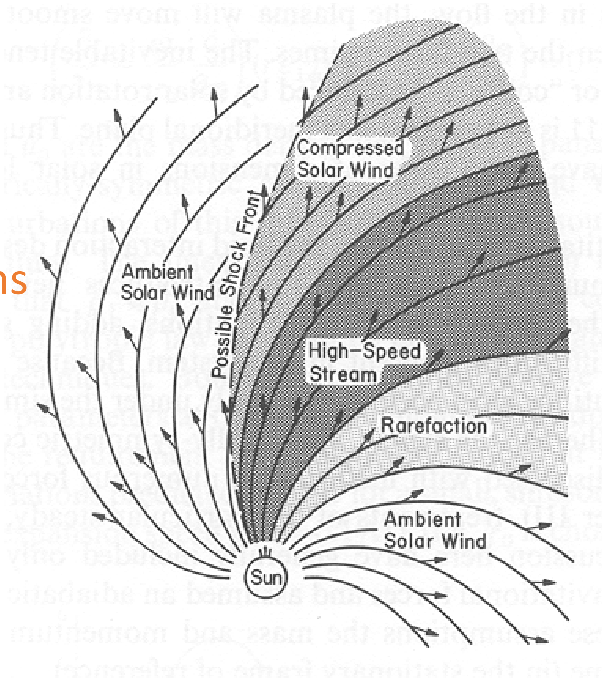


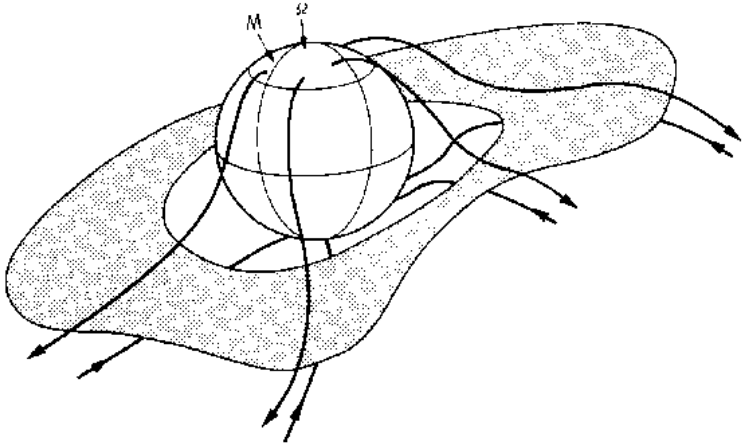
FE IX/X lines, TRACE

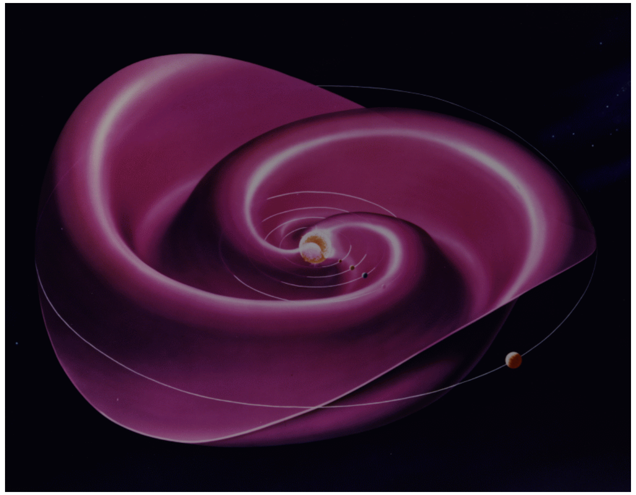
Large scale features of the solar wind



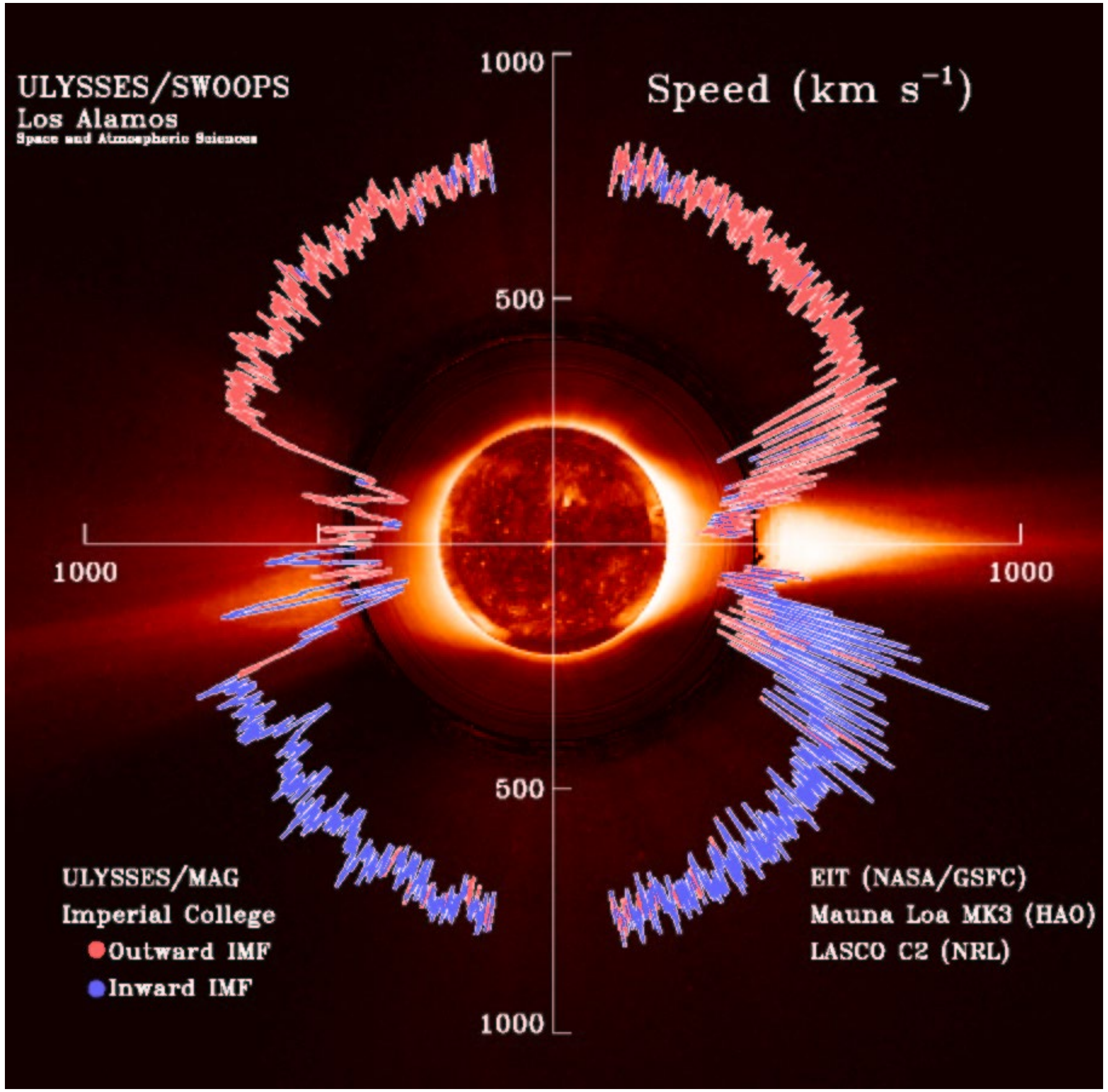
- Plasma outflow, spiral magnetic field
- High and low speed streams
- North south distorted magnetic dipole
- Wavy, equatorial current sheet







Large scale features of the Solar Wind: *Ulysses*



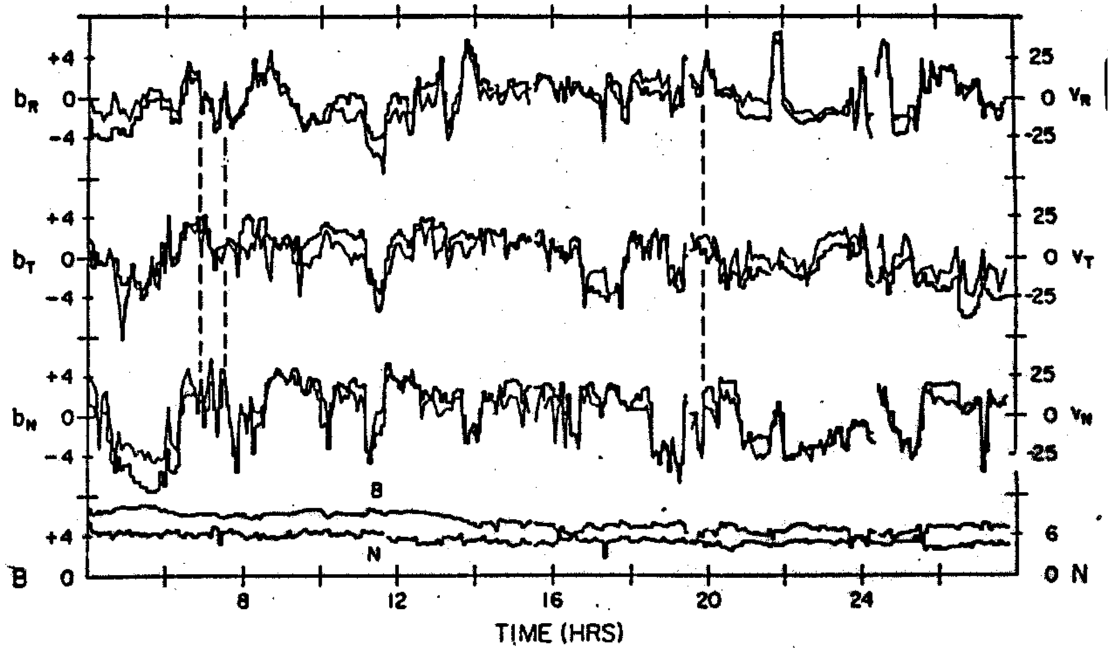
- High latitude
  - Fast
  - Hot
  - steady
  - Comes from coronal holes
- Low latitude
  - slow
  - "cooler" (40,000 K @ 1 AU)
  - nonsteady
  - Comes from streamer belt

McComas et al, GRL, 1995

The solar wind is turbulent

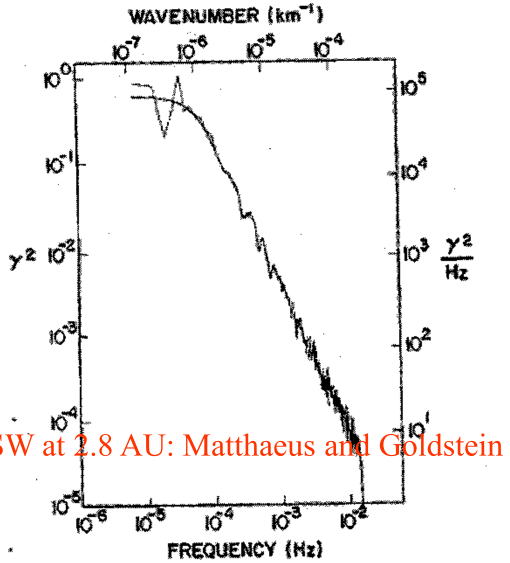
- Fluctuations in velocity and magnetic field are irregular, not "reproducible," broad-band in space and time
- Indications of turbulence properties and wave-like properties

Mariner 2 data



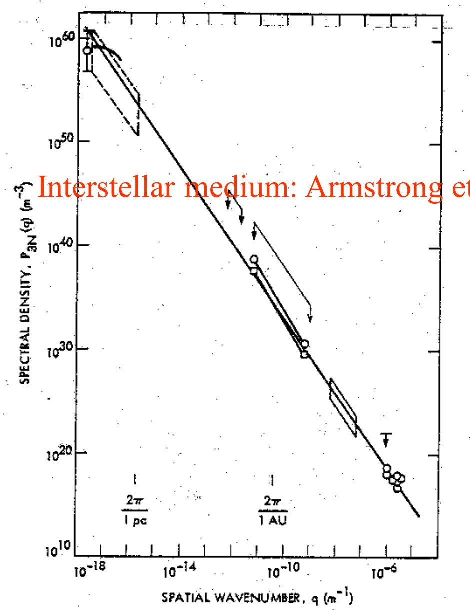
Belcher and Davis, JGR, 1972

"Powerlaws everywhere"→ Broadband self-similar spectra are a signature of cascade

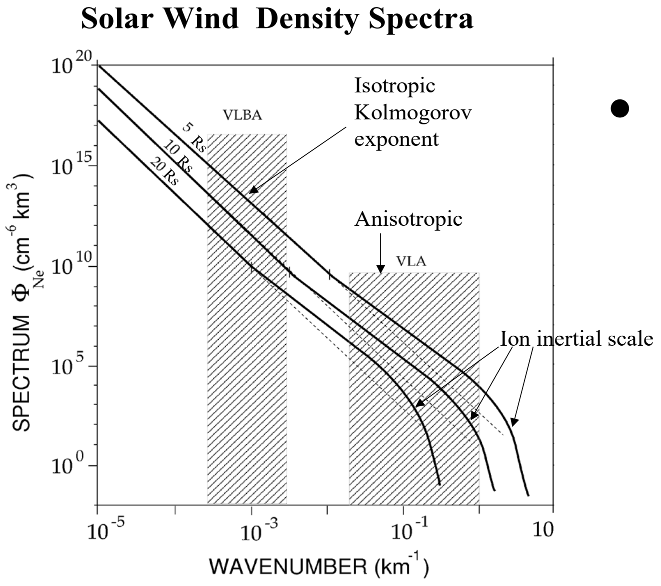


SW at 2.8 AU: Matthaeus and Goldstein

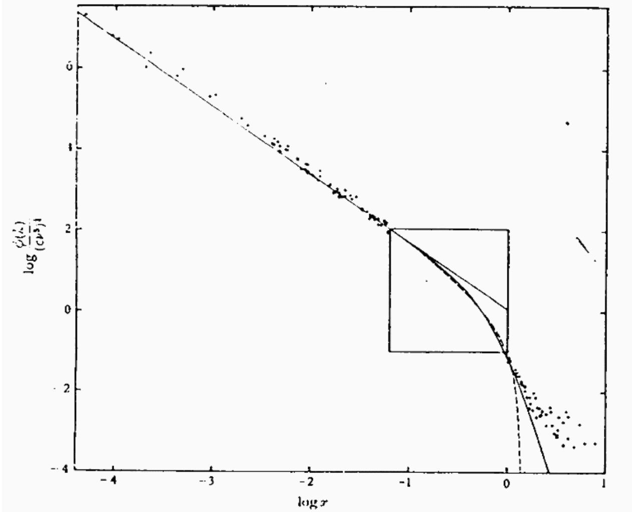
- Solar wind
- Corona
- Diffuse ISM
- Geophysical flows



Interstellar medium: Armstrong et al



Coronal scintillation results (Harmon and Coles)



Tidal channel: Grant, Stewart and Moilliet

II. What does turbulence do, and why multipoint measurements?

What is turbulence ?

- Can we define turbulence? …give it a try:
  → Turbulence is the **complex dynamics of a fluid or plasma** that is due to **nonlinear interactions** involving a (probably wide) **range of space and time scales**.

- The details of turbulent dynamics exhibit a **sensitivity to small changes in initial or boundary data**. Turbulence shares this feature with its close relative, chaos, which often involves only a few degrees of freedom. In some contexts, distinction is made between "deterministic chaos" and "stochastic behavior", but (in my view) turbulence in a system with many degrees of freedom blurs this distinction. (Mathematicians have fun with these fine distinction!).

- Generally speaking turbulence requires a **statistical description**, even it it is formally deterministic.

→ one introduces vocabulary that includes probability distributions (PDFs) , ensemble averages, and moments, including correlation in space ands timeat various orders , increments, spectra, etc

→ the most well-developed cases are special ones: homogeneous turb; time statiionary turb; incompressible turb.; and vast progress has been made in these special cases . Caution: turbulence in the real world likely violates these idealizations, which nonetheless are useful in extracting physical insight.

Some distinctive effects of “turbulence”

- Cross scale couplings
- Appearance of “cascade”
- Enhanced dissipation/heating at small scales, governed by large scale dynamics (GI Taylor, von Karman)
- In large systems lacking preferred scales over some substantial range, a scale invariant powerlaw distribution of excitations (energy?) –Kolmogorov
- Enhanced transport (of potentially many kinds)
- Mixing & complex Lagrangian trajectories (of fluid elements, particles, field lines…)
- Even when means are well defined, there may be large fluctuations (as in dissipation). This leads to nongaussian effects, intermittency and extreme “events”

Why multipoint measurements?

- Space plasma are inherently three dimensional
- Turbulence in particular is inherently three dimensional
- Single point diagnostics (such as single spacecraft) can only provide 3D information by adopting additional assumptions – e.g., isotropy, Taylor hypothesis, etc
- To measure in situ the required fundamental turbulence statistcs requires multiple probes separated in space
- Space and time correlations are distinct and both important in understanding turbulence. Multiple probes recording in time enables separating behavior inspace & time.
- This line of inquiry reveals fundamental physics, not just phenomenology.

Some references

Report of the NASA Plasma Turbulence Explorer Study Group

September 1980

National Aeronautics and Space Administration

Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California

White Paper submitted to: Decadal Survey for Solar and Space Physics (Heliophysics) 2024-2033

The essential role of multi-point measurements in investigations of turbulence, three-dimensional structure, and dynamics: the solar wind beyond single scale and the Taylor Hypothesis

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(2019)
https://arxiv.org/abs/1903.06890

JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 107, NO. A11, 1384, doi:10.1029/2001JA005088, 2002

Four-point Cluster application of magnetic field analysis tools: The Curlometer

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III. Basic statistical descriptions in terms of correlation functions

Basic turbulence properties

- Consider turbulent fields, e.g., v(x,t), b(x,t)
- Can separate mean fields and fluctuations, e.g., $\mathrm{b} = \mathrm{B} - \langle \mathrm{B} \rangle$
- Reynolds (ensemble)
- Ergodic property: time average over period T converge in the mean to the ensemble mean.
- Sometimes the average does not depend on the origin of x and/or t

  then <B> = B0 is spatially uniform and/or time independent

  i.e., the mean is statistically homogeneous or (time-)stationary

Hydrodynamic turbulence: *a la GI Taylor, Batchelor, Kolmogorov….*

- The most basic quantities of interest in describing homogeneous fluid turbulence are statistical quantities –
  - Averages (variances)
  - Correlation functions
    - (2 point, single time)

$$R_{vv}(\mathbf{r}) = \langle \mathbf{v}(\mathbf{x}) \cdot \mathbf{v}(\mathbf{x} + \mathbf{r}) \rangle$$

Hydrodynamic turbulence: *a la GI Taylor, Batchelor, Kolmogorov….*

- Deeper insight is obtained by computing two point, two time corelations: treat space & time separately

  - For turbulent velocity v

$$R(\mathbf{r}, \tau) = < \mathbf{v}(\mathbf{x}, t) \,.\, \mathbf{v}(\mathbf{x} + \mathbf{r}, t+\tau) >$$

2 point, single time $R(\mathbf{r}, 0)$

1 point, two-time $R(0, t)$

Correlation functions

- Second order (covariance) of a scalar varying only in time
- $c(t) = C(t) - \langle C \rangle$
- Reynolds (ensemble) average $\langle \ldots \rangle$
- $R(\tau) = \langle c(t)\, c(t + tau) \rangle$ is "stationary" if R does not depend on time t, i.e., independent of origin of time

- The spectrum of $c(t)$ is the Fourier transform of $R(t)$:

$$S(\omega) = \frac{1}{2\pi} \int d\tau\, R(\tau) \exp\{i\, \omega\, \tau\}$$

Various methods exist for approximate determination of R or S from data.

for periodic data in a finite domain, special relations exist, e.g.,
it can be shown that

$$S_{\{periodic\}}(\omega) \sim \left\langle \left| C_{\{periodic\}}(\tau) \right|^2 \right\rangle$$

Homework (optional): Read up on this relation, and try to prove it. HINT: you should first recall the relationship between Fourier series amplitudes (finite domain) and Fourier transforms (infinite domain)

Wavenumber frequency decomposition

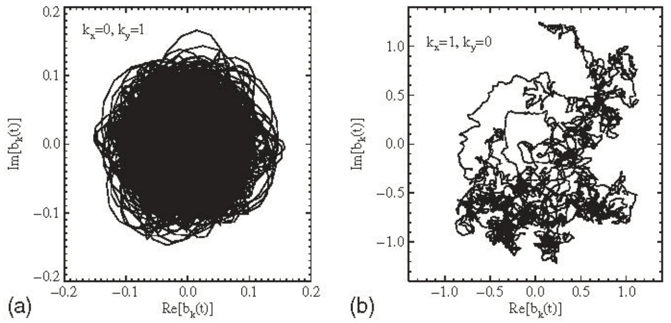
$$R_{\alpha\beta}(\mathbf{r},\tau) \equiv \langle b_\alpha(\mathbf{x},t) b_\beta(\mathbf{x}+\mathbf{r},t+\tau)\rangle$$
$$= \int d^3k \; d\omega \; S_{\alpha\beta}(\mathbf{k},\omega) e^{i\mathbf{k}\cdot\mathbf{r}} e^{i\omega\tau}.$$

One might think of this as a generalization of a dispersion relation…

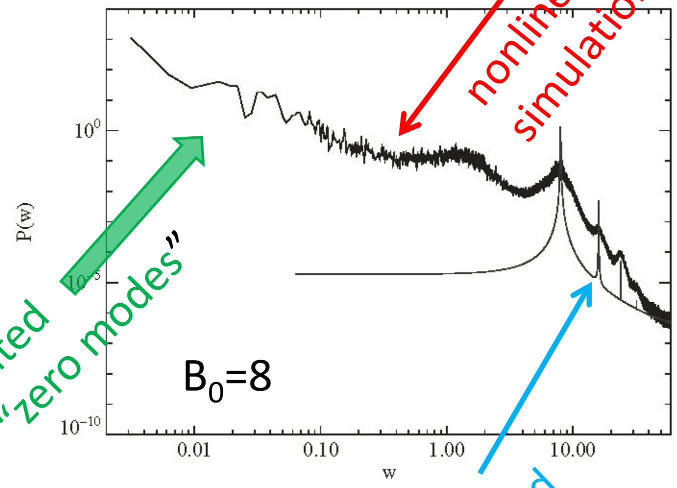
Except that in turbulence
each wavenumber has broad-band frequency content
Often dominated by very low ("zero") frequencies!

Numerical experiments on MHD Turbulence with mean field:

behavior of a Fourier mode in time, from simulation



Eulerian frequency spectrum: transform of one point two time correlation fn.



maximum of ~13% of energy in linear modes – that occurs when dB/B0 ~ 1/2

Dmitruk & Matthaeus, 2009

Magnetic field space-time correlation

- Switch from velocity **v** to magnetic field **b**
- Two point, two time correlation of fluctuating **b** with components b_j

$$R_{ij}(\boldsymbol{r},\tau) = \langle\ b_i\,(\boldsymbol{x}+\boldsymbol{r}, t+\tau)\, b_j(\boldsymbol{x},t)\ \rangle$$

define wavenumber spectrum

$S(\boldsymbol{k})$ (e.g., K41)

$$R(\boldsymbol{r}) \equiv R_{ii}(\boldsymbol{r},0) = \int d^3k\, S(\boldsymbol{k})\ \exp\{i\, \boldsymbol{k}\cdot\boldsymbol{r}\}$$

define Eulerian frequency spectrum

$$F(\omega) \equiv\ (2\pi)^{-1}\int d\tau\, R(0,\tau)\exp(-\, i\, \omega\tau)$$

$$=(2\pi)^{-1}\int d^3k \int d\tau\, S\big(\boldsymbol{k}\big)\Gamma\big(\boldsymbol{k},\tau\big)\ \exp\big\{-\, i\, \omega\tau\big\}$$

IV. Second order statistics- correlations and spectra

Second order spatial correlation (now, mostly magnetic field **b**)

- Two pointcorrelation functions
- Their associated spectra (fourier transforms)

$$R_{bb}(\mathbf{r}) = \langle \mathbf{b}(\mathbf{x}) \cdot \mathbf{b}(\mathbf{x} + \mathbf{r}) \rangle$$

Often use similarity variables (von Karman): turbulence energy, correlation scale

$R(\mathbf{r}) \rightarrow Z^2 \; R(\;\mathbf{r} \; / \; \lambda\;)$

Single s/c background: Taylor hypothesis:
a simple relation between time and space correlations

Space-time correlation

$$R_{bb}(\mathbf{r}, \tau) = \langle \mathbf{b}(\mathbf{x}, \mathbf{t}) \cdot \mathbf{b}(\mathbf{x} + \mathbf{r}, t + \tau) \rangle$$

suppose fluctuation
moves undistorted along
fast flow U

→

$$R_{bb}(\mathbf{r} + \mathbf{U}t, t) = R_{bb}(\mathbf{r}, 0)$$

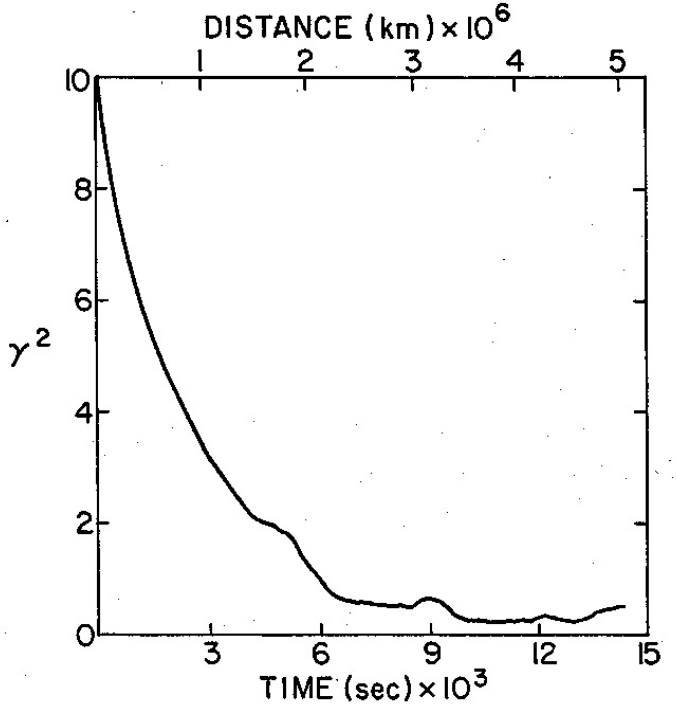
Then measured Eulerian correlation at 1 s/c gives
2-point single time correlation through

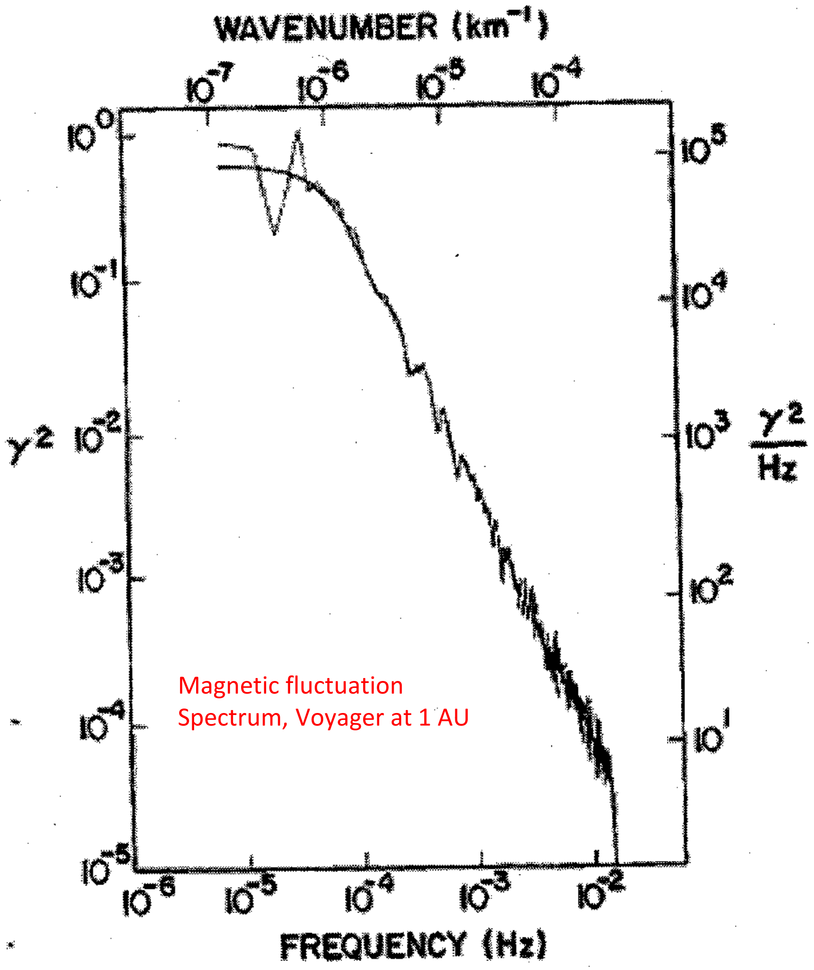
$$R_{bb}(\mathbf{r} = -\mathbf{U}t, 0) \approx R_{bb}(0, t)$$

but this mixes space- and time- decorrelation, and while
useful, needs to be verified (as an approximation) and further
studied to unravel the distinct decorrelation effects

MHD scale turbulence in the solar wind (obtained with Taylor “frozen in” approximation

- Powerlaw spectra →cascade
- spectrum, ←→ correlation function

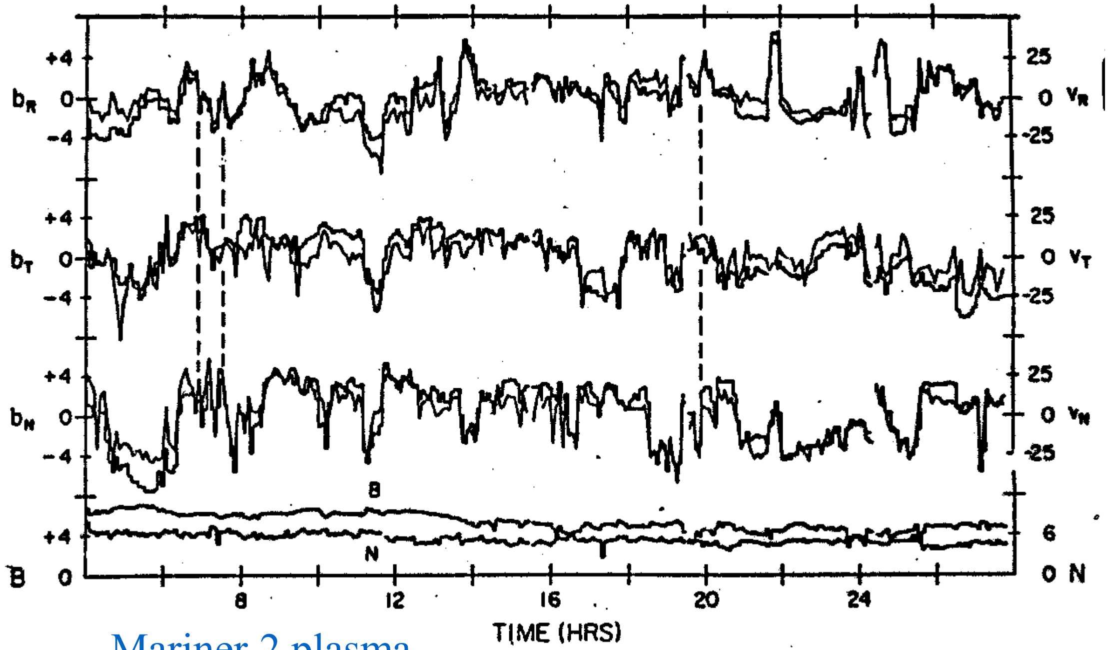




Matthaeus + Goldstein JGR 1982

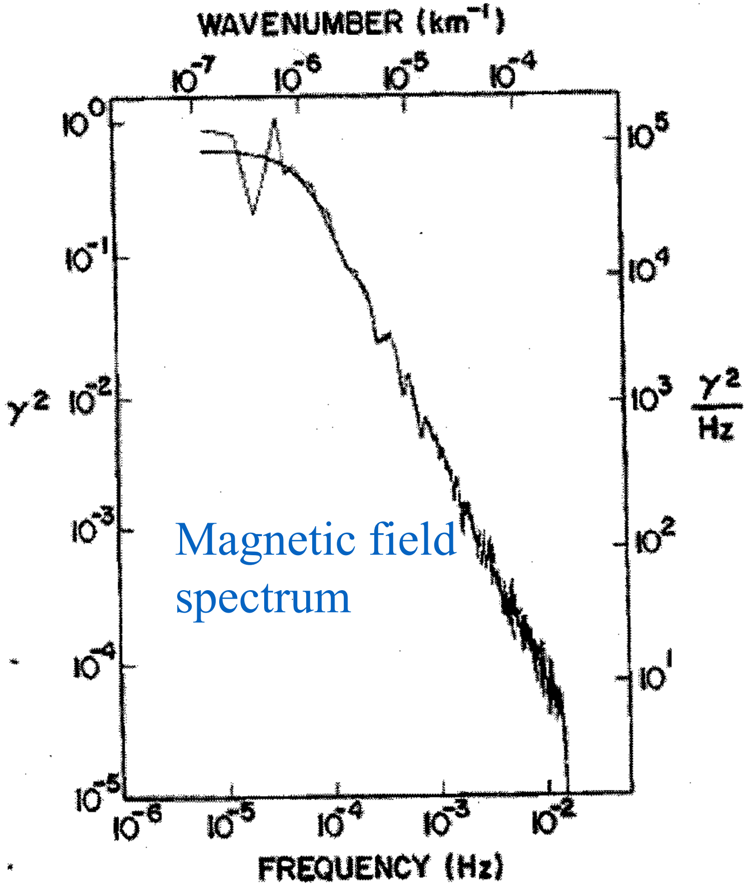
Solar wind: indications of both **turbulence** and **wave-like** properties:

- Powerlaws
- "Alfvenic fluctuations"



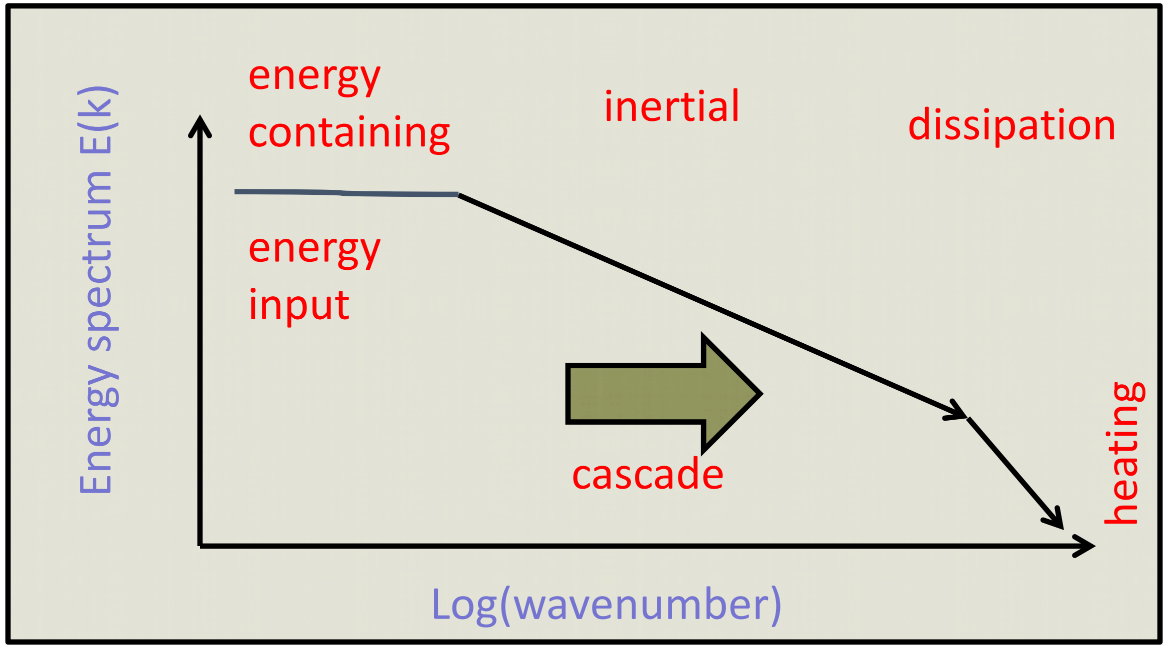
Mariner 2 plasma and magnetic field data

Belcher and Davis, JGR, 1972



SW at 2.8 AU: Matthaeus and Goldstein

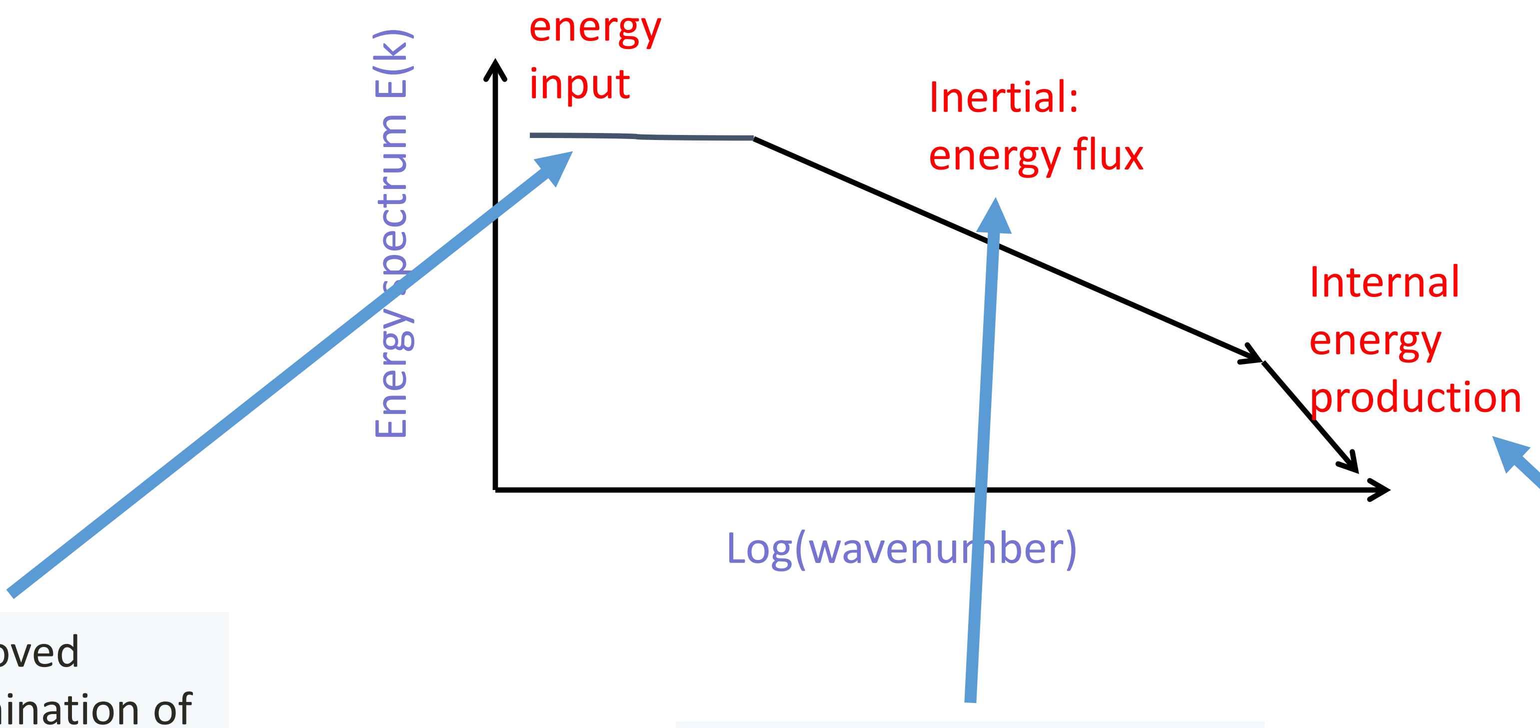
“standard” turbulence spectrum



- **Heating:** increase in random kinetic energy
- **Dissipation:** conversion of (collective) fluid degrees of freedom into motions into kinetic degrees of freedom
- **Entropy** increase: irreversible heating

V. Multi point/ multi-spacecraft measurements!

Multispacecraft enables measurements of all standard aspects of the cascade



…improved determination of correlation scale: **Reviewed below**

… improved measurement of energy flux across scales: **See LPDE- Servidio lectures**

…pressure strain measurement *REQUIRES* multipoint measurements. **See pressure-strain - Bandyopadhyay lectures**

Physics to be revealed by multi spacecraft correlations

- Space (correlation length, Taylor scale) - cascade
- Physics of anisotropy – mean magnetic field, etc
- Time (eulerian correlation time) - diffusion
- Space-time (propagator, scale dependent decorrelation) – the nature of turbulence couplings

What multi s/c can tell us: first steps

- Frozen-in flow (predictability)
- Spatial correlations in $(r_{\parallel}, r_{\perp})$
- Infer the Eulerian (two time, 1 pt) correlation

Plan of action:

I) develop a methodology

make use lessons from single spacecraft studies
of stationarity, turbulence PDFs, variability, etc

II) first application: magnetic field two point single time correlation near 1 AU

Ensemble→ mean + fluctuation

$$\mathbf{b} = \mathbf{B} - \langle \mathbf{B} \rangle$$

Correlation function
(F.T. of spectrum)

$$R_{bb}(\mathbf{r}) = \langle \mathbf{b}(\mathbf{x}) \cdot \mathbf{b}(\mathbf{x}+\mathbf{r}) \rangle$$

Matthaeus+PRL 95, 231101 (2005)

Mean in interval *I*

$$\mathbf{b}_S^I = \mathbf{B}_S^I - \mathbf{B_0}_S^I$$

Energy interval *I*

$$E^I_{b/S1,S2} = \frac{< (\mathbf{b}^I_{S1})^2 > + < (\mathbf{b}^I_{S2})^2 >}{2}.$$

Structure function estimate interval *I*

$$Sbb(r_{S1,S2})^I = \frac{\langle(\mathbf{b}^I_{S1} - \mathbf{b}^I_{S2})^2\rangle}{2E^I_{b/S1,S2}}.$$

Correlation function estimate interval *I*

$$Rbb(r_{S1,S2})^I = 1 - Sbb(r_{S1,S2})^I = \frac{\langle \mathbf{b_{S1}}^I \cdot \mathbf{b_{S2}}^I \rangle}{E^I_{b/S1,S2}}$$

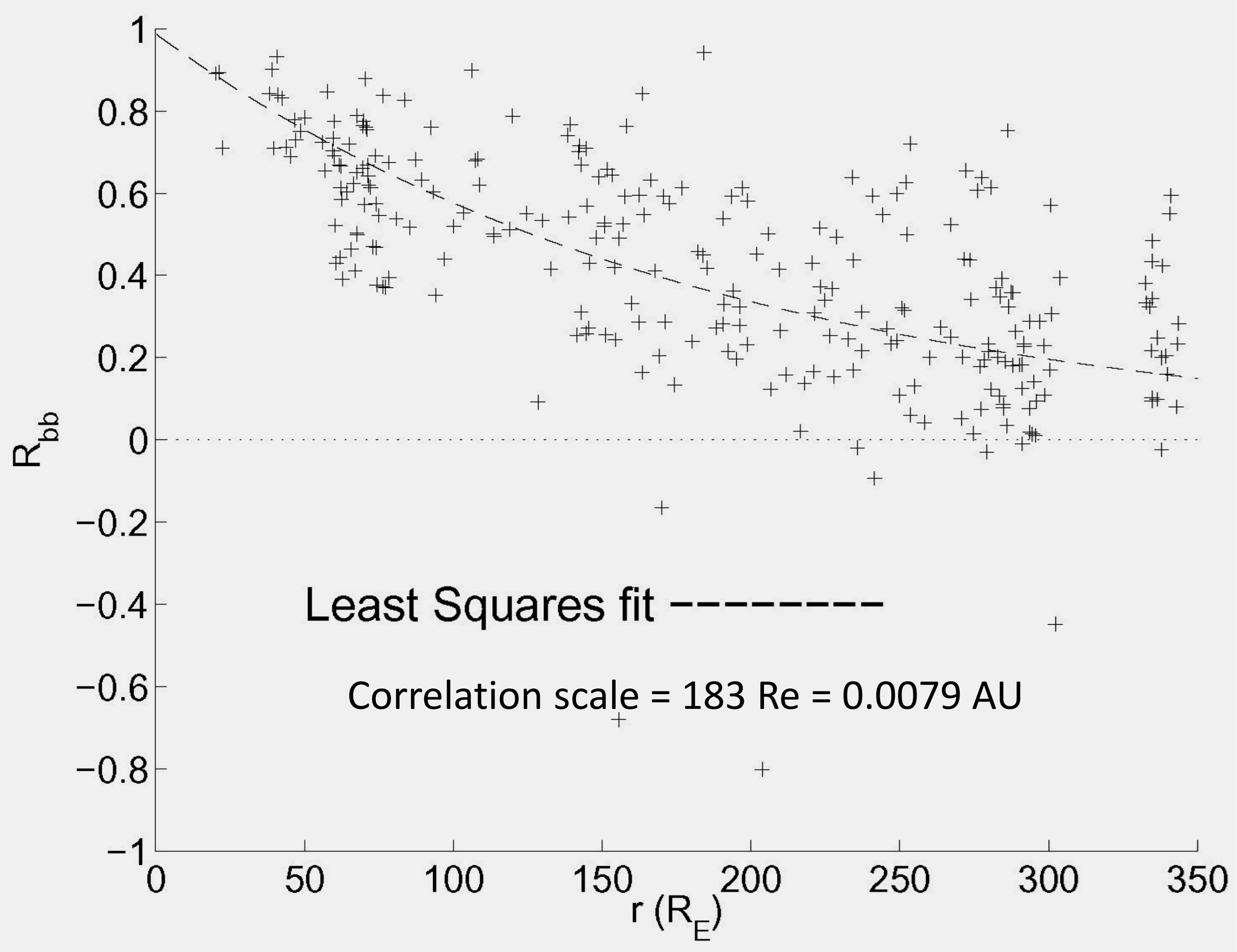
→ Estimates are variance-normalized (reduces source variability effects)

VI. Multi-s/c: Second order spatial correlation

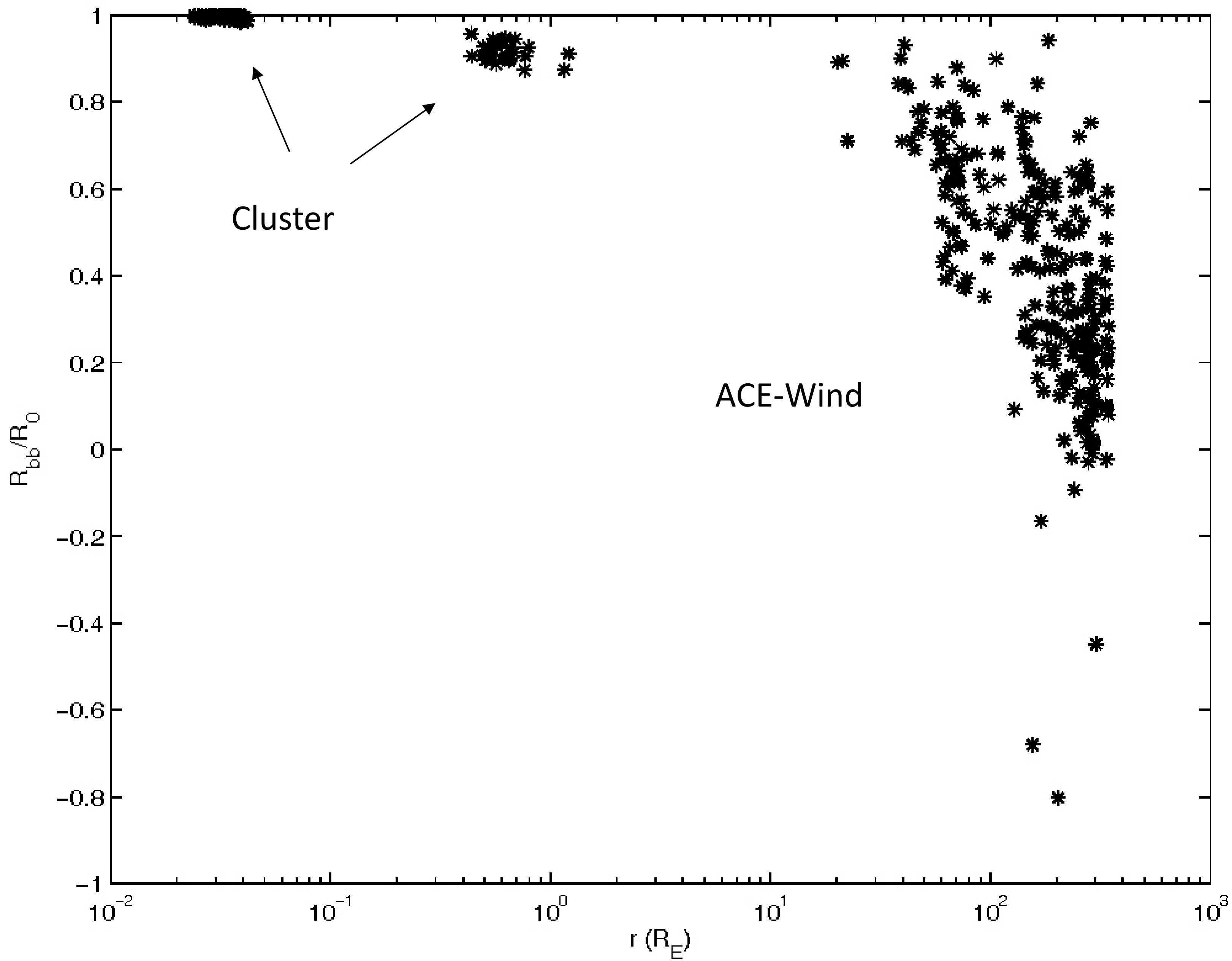
ACE-WIND correlations: 2^{nd} order spatial corr.

- 1 minute samples
- 1 day data intervals
- Use single separation for each interval
- Subtract mean field for each interval
- correlation coefficients for each interval
- normalize correlation function estimates by observed variances

Normalized multi- s/c correlation function



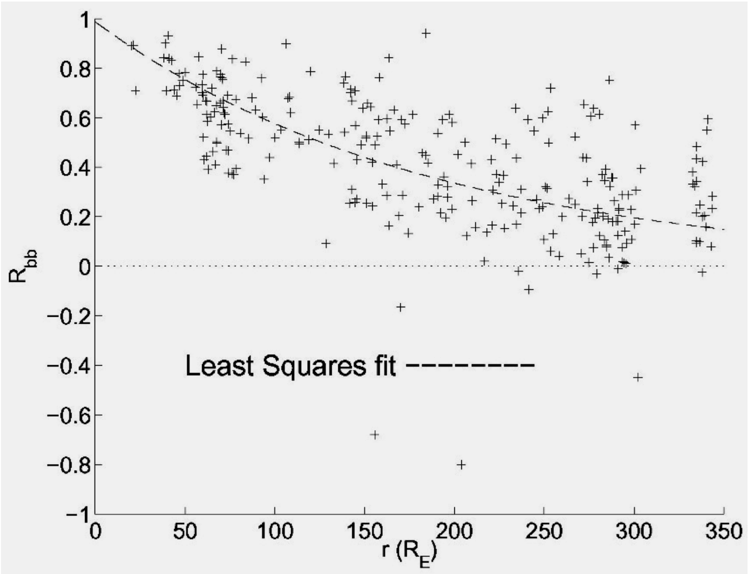
Groups 1, 2, 3

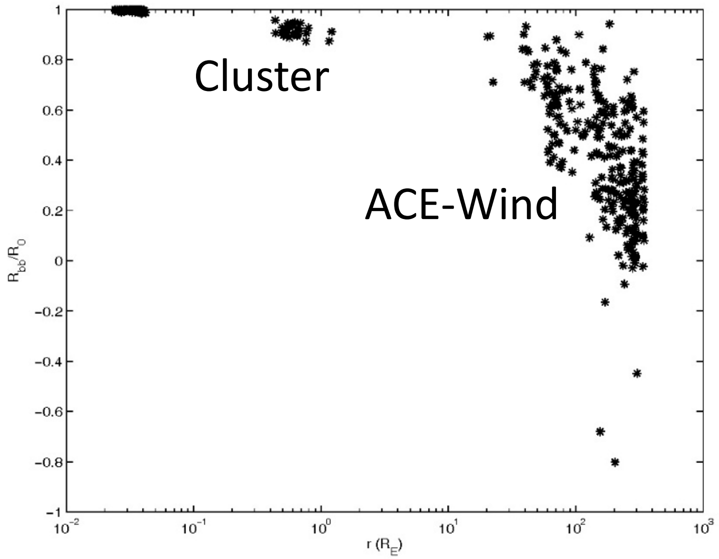


Cluster
ACE-Wind
R_{bb}/R_0
r (R_E)
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}

Correlation function/correlation length from 2 s/c measurements

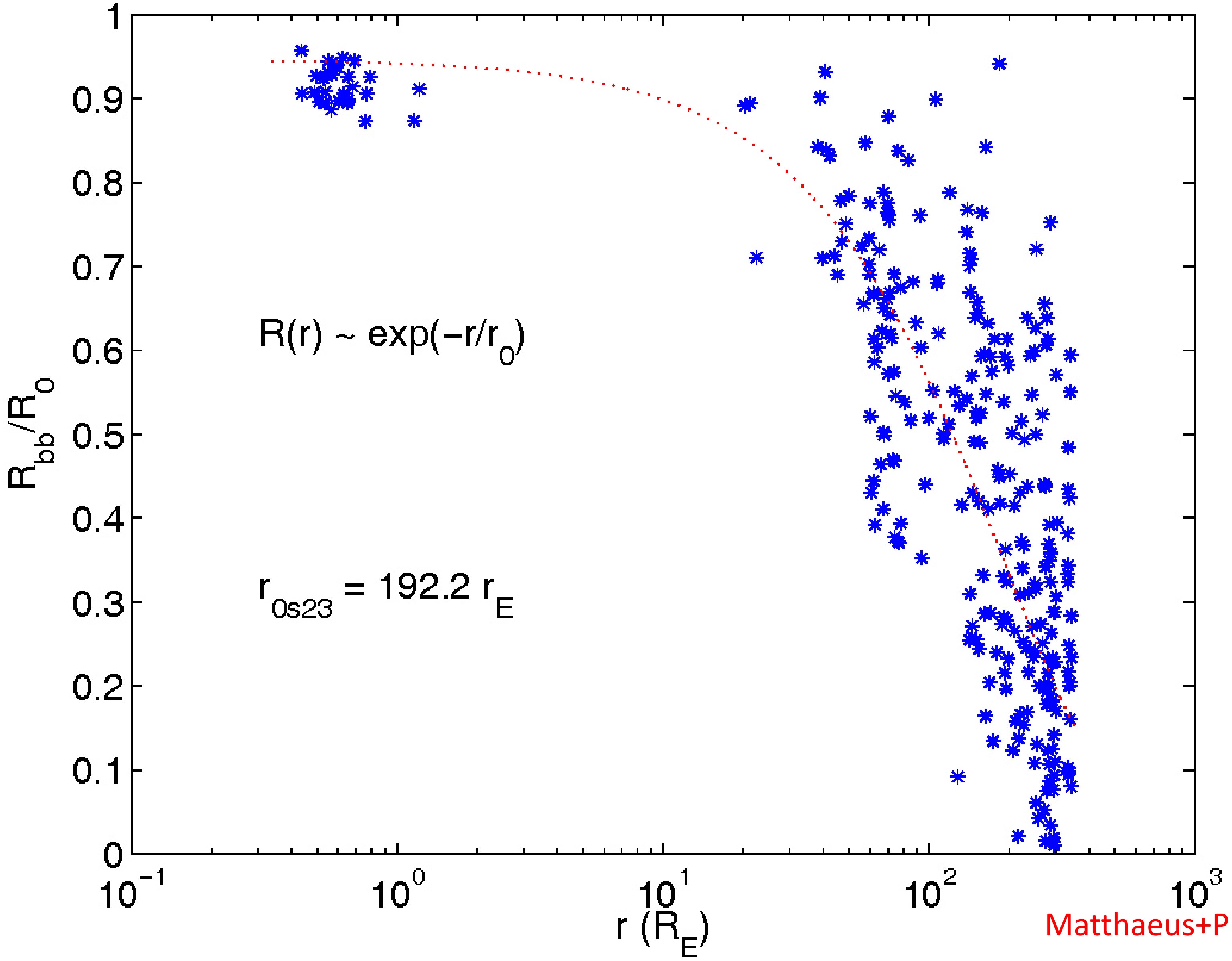
- ACE-WIND correlations
- 1 minute samples
- 1 day data intervals
- Use single separation for each interval
- Subtract mean field for each interval
- correlation coefficients for each interval
- normalize correlation function estimates by observed variances





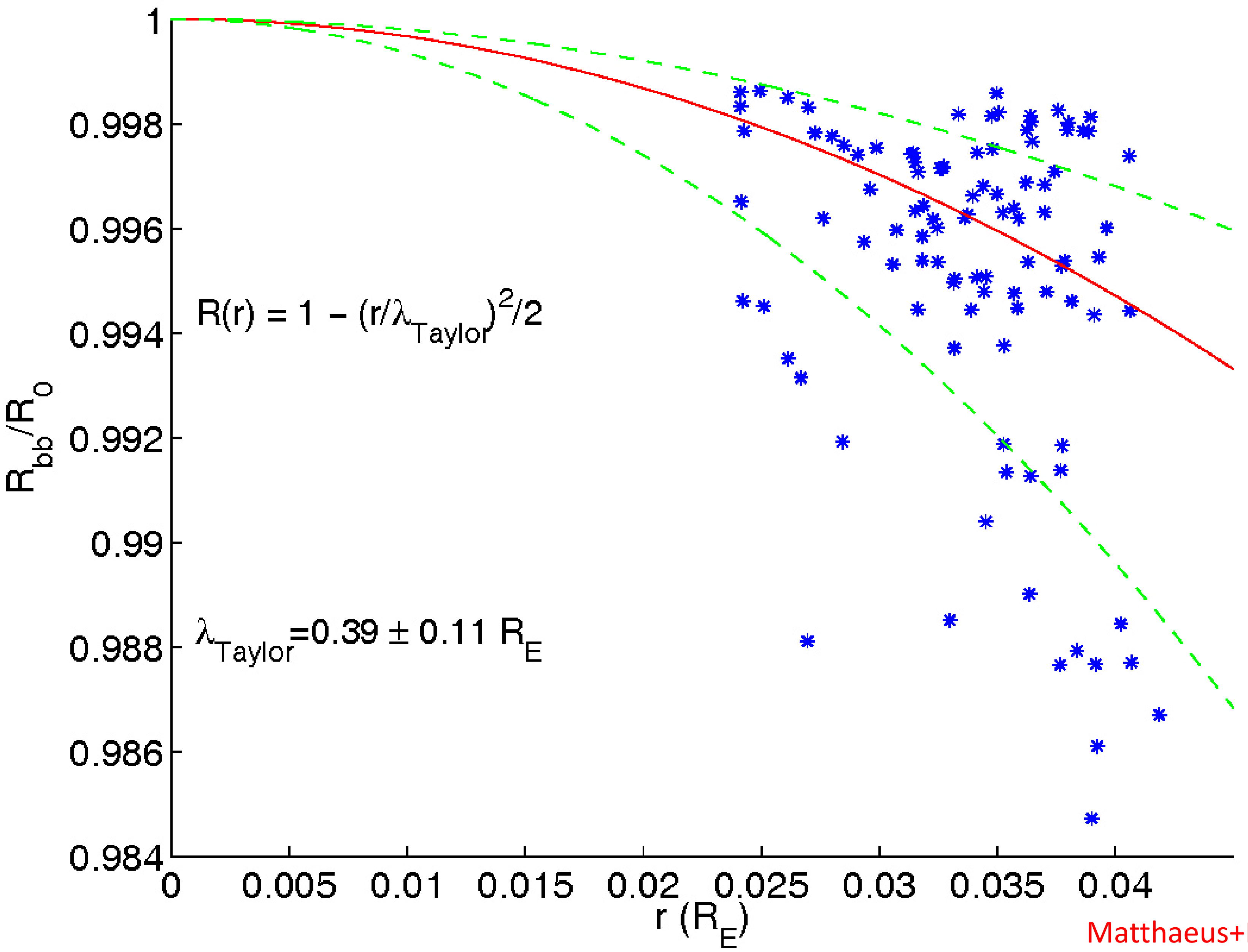
Matthaeus+PRL 95, 231101 (2005)

Exponential fit to determine outer scale



Matthaeus+PRL 95, 231101 (2005)

Parabolic fits (group 1) to establish inner (Taylor scale)



Matthaeus+PRL 95, 231101 (2005)

Conclusions: multi spacecraft two point, single time correlations of SW turbulence

- correlation (outer) scale

  L_c = 183 – 192 Re (~ 0.008 au)

- inner (Taylor) scale

  $\lambda_{Taylor} = 0.39$ Re (~1.6×10^{-5} au)

So, effective Reynolds number of SW turbulence is

$(L_c/\lambda_T)^2 \sim 230{,}000$

- Multi s/c correlations appear to be confirming Maltese cross
- Somewhat larger Lc in single s/c analyses probably due to low frequency time variability (1/f noise) which does NOT influence the present multi s/c analysis

Matthaeus+PRL 95, 231101 (2005)

VII. Multi-s/c Second order time (Eulerian) correlation

Eulerian time correlation

$$F(\tau) = \langle \mathbf{b}(\mathbf{x}, t) \cdot \mathbf{b}(\mathbf{x}, t + \tau) \rangle$$
$$= \int d\omega \hat{F}(\omega) \exp i\omega\tau .$$

Abbreviating $S(\mathbf{k}) \equiv S_{\alpha\alpha}(\mathbf{k})$, we may factorize the time dependence as

$$F(\tau) = \int d^3k S(\mathbf{k}) \Gamma(\mathbf{k}, \tau). \tag{4}$$

The quantity $\Gamma(\mathbf{k}, \tau)$ fixes the rate of decorrelation in time.

Eulerian cont’d

Matthaeus+ ApJ, 721, L10 (2010)

Two s/c separated by more than
10^6 km
give similar 2 point correlations.
(at least in this case)

What can we learn about this
by looking at statistics of many intervals?

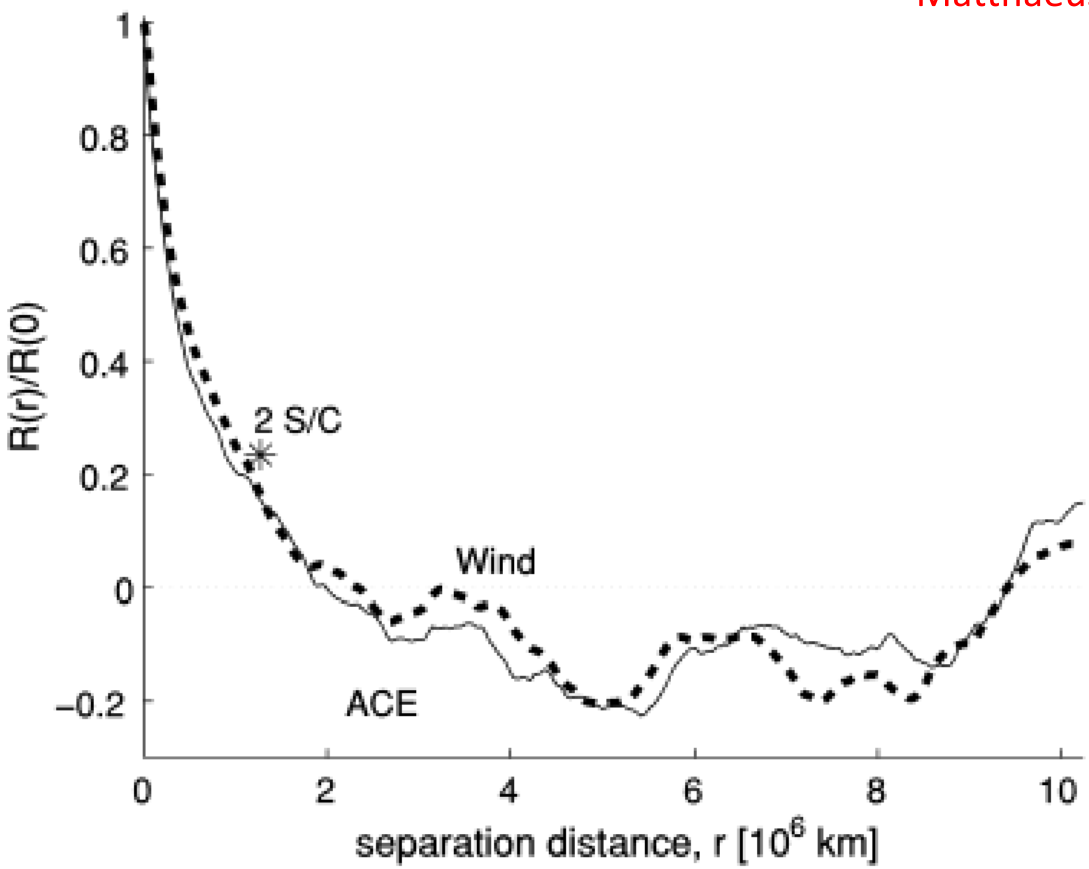


Figure 2. Magnetic correlation functions vs. distance for one 24 hr interval of *ACE* and *Wind* magnetic field data in the solar wind near Earth orbit (1999 October 4, 00–24 UT). Each spacecraft provides a correlation estimate using frozen-in flow (solid and dotted lines). *Wind–ACE* cross correlation provides a single correlation estimate at the spacecraft separation (asterisk). To the extent that these estimates agree, the Taylor hypothesis is exact. Analysis of the small differences in many such observations is used here to estimate the average Eulerian decorrelation rate.

Eulerian (cont'd)

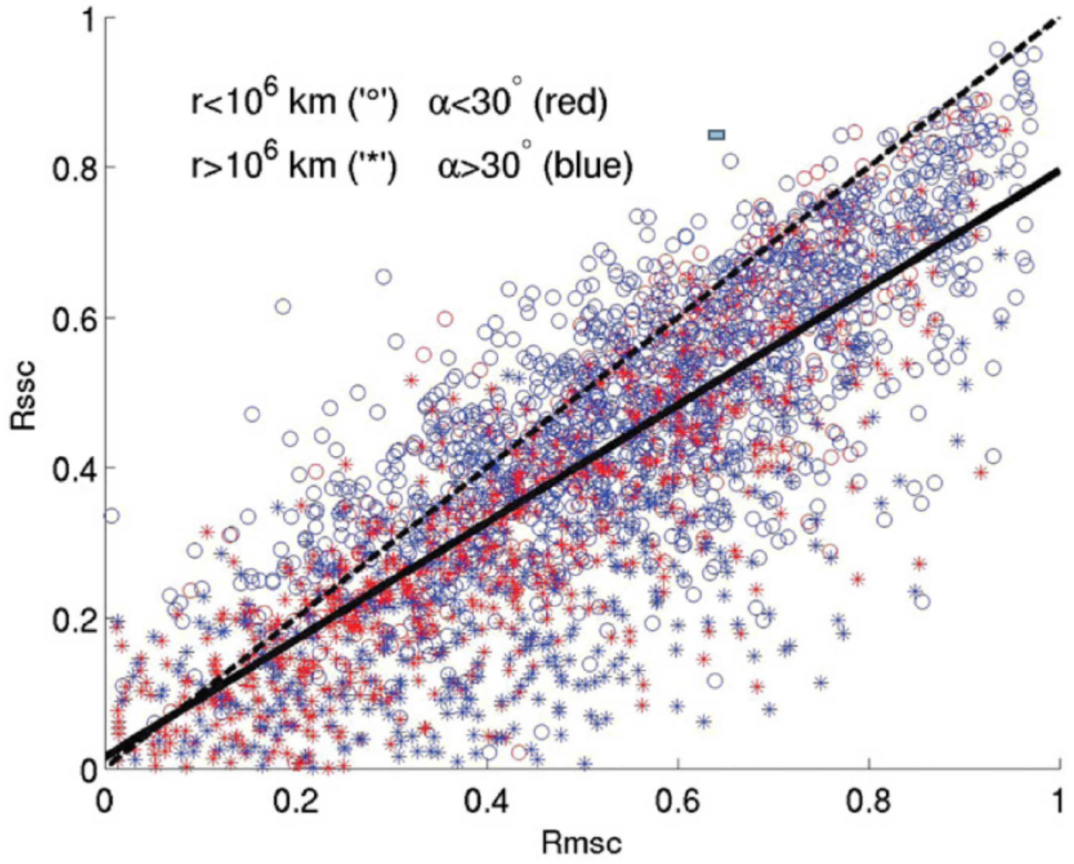


Figure 3. Scatter plot of single-spacecraft correlation (R_{ssc}) vs. two-spacecraft correlation (R_{msc}), at a common spatial lag, where 2584 cases from *ACE* and *Wind* data are used. The line $R_{ssc} = R_{msc}$ (dashed) and a linear least-squares line (solid) are shown for reference. Data are sorted according to angle α subtended by spacecraft separation and radial direction.

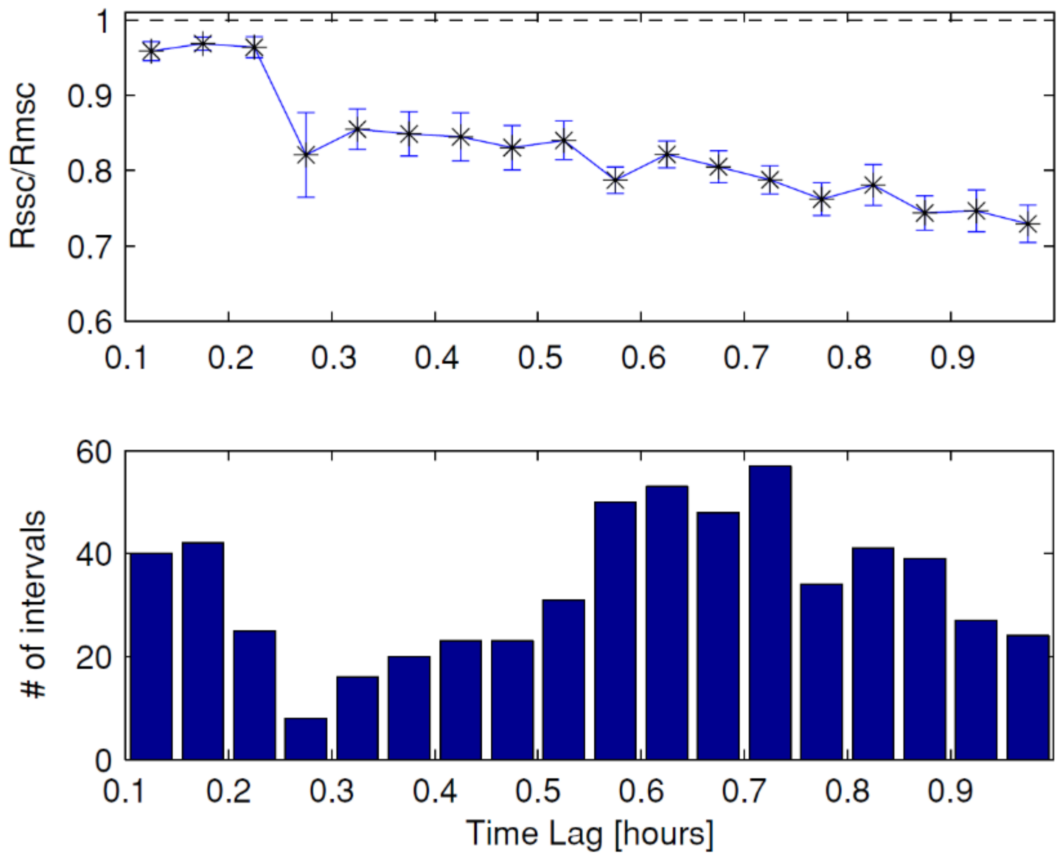


Figure 4. Top: estimated normalized Eulerian correlation $F(\tau)/\langle|\mathbf{b}|^2\rangle$ from ratio of single-spacecraft correlation estimates to two-spacecraft correlation estimates binned by time lags. Symbols are bin averages. Bottom: the number of intervals in each bin. For all intervals spacecraft are within 30° of radial alignment.

Eulerian: separability assumption yields a simple result

the Taylor approximation. Specifically, we make the simplifying assumption that $\Gamma(\mathbf{k}, \tau) = \Gamma(\tau)$ independent of $\mathbf{k}$. Furthermore, we adopt the functional form

$$\Gamma(\tau) = e^{-\tau/\tau_c} \tag{6}$$

with a single Eulerian decorrelation timescale τ_c to be determined by the procedure. Although not a fundamental relation (see, e.g., Chen & Kraichnan 1989; Zhou et al. 2004; Shalchi et al. 2006; Shalchi 2008), the above ansatz leads to a convenient separability of the space and time dependence, and an improvement over the frozen-in flow approximation. In particular, with this simple choice, the single-time two-point correlation and single-point two-time correlation are related by

$$R(r, \tau) = R(r, 0)\Gamma(\tau) = R(r, 0)e^{-\tau/\tau_c}. \tag{7}$$

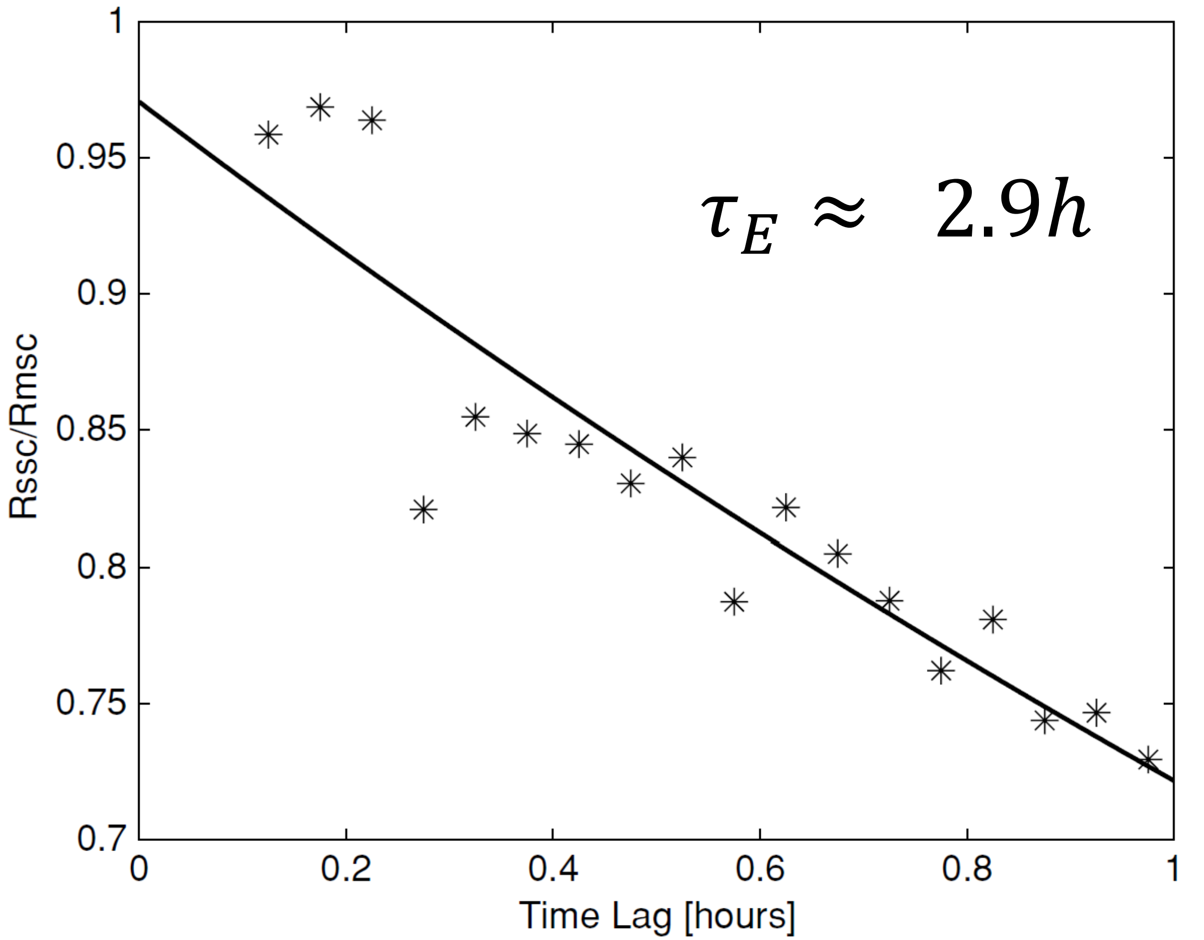


Figure 5. Estimated normalized Eulerian correlation $F(\tau)/\langle|\mathbf{b}|^2\rangle$. An exponential fit is shown with a decorrelation time of 2.9 hr.

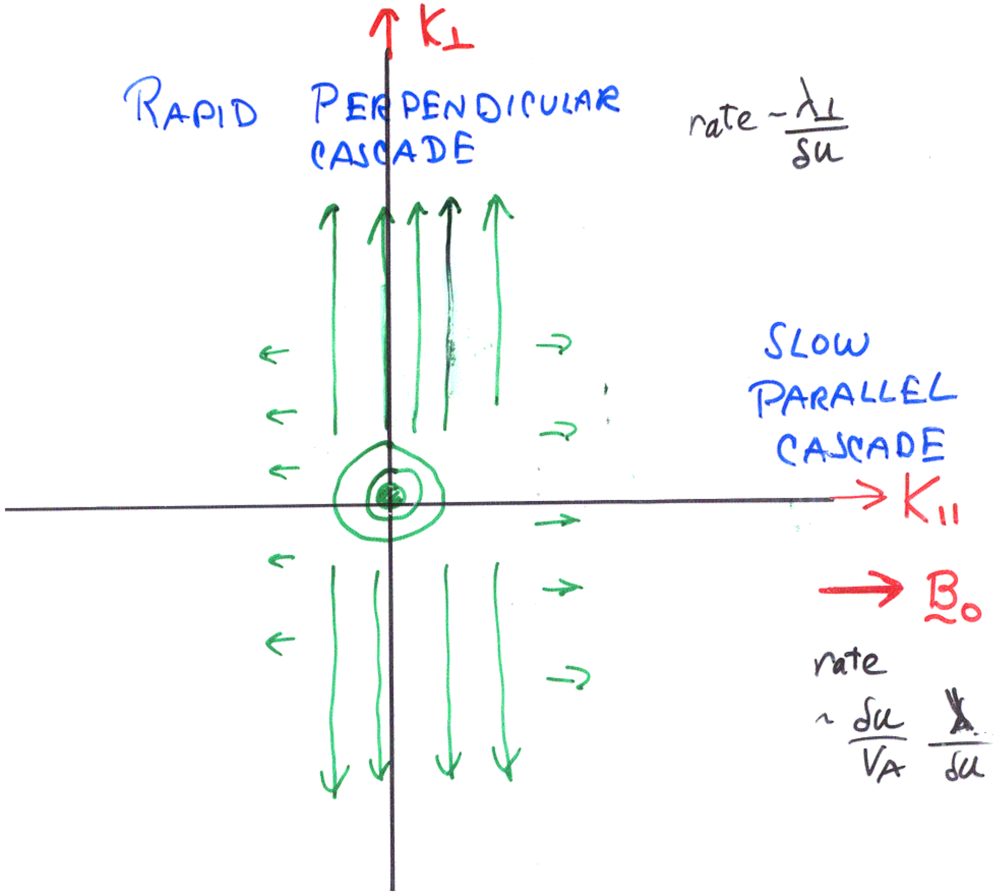
VIII. Anisotropy with multi-spacecraft

Correlation/spectral **anisotropy**

- Many inferences about turbulence and various indications of anisotropy
- Single spacecraft observations (frozen-in flow)
- Particle scattering
- Laboratory expts
- Simulations
- But, to confirm requires (at least) with two-point statistical measurements

- A robust result -- Generation of larger gradients perpendicular to the magnetic field direction

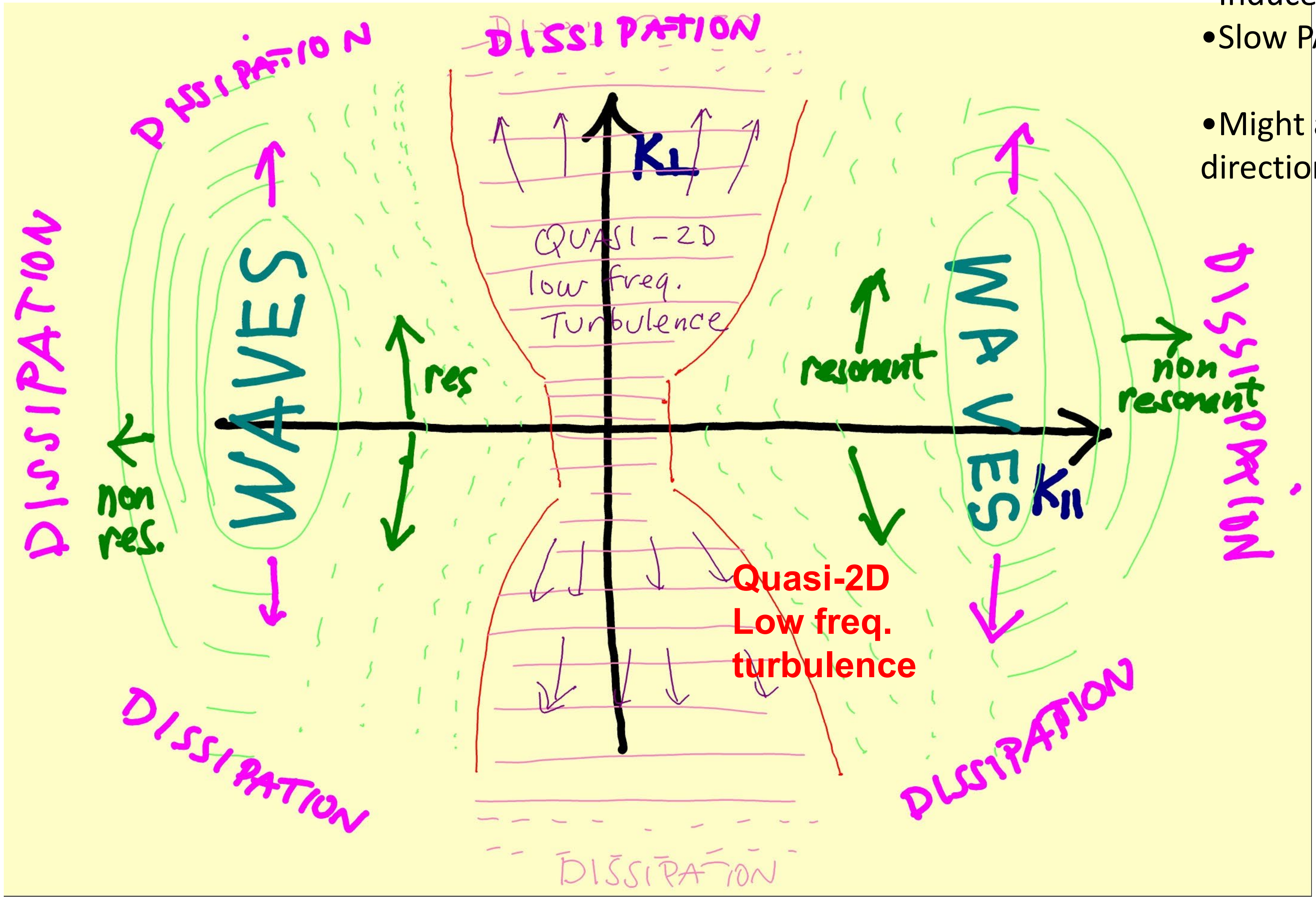
•Laboratory devices
(Zeta, Macrotor)
•Theory of reduced MHD
(Strauss, 1978; Montgomery, 1982)
- Simulations (Shebalin+ 1983; Oughton+ 1994)

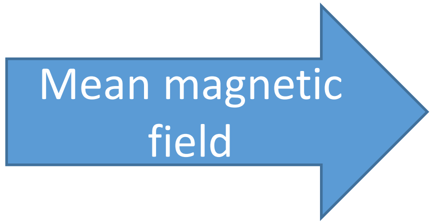


Anisotropic cascade + waves, relative to B_0

- Rapid, hydrolike PERP transfer
- Induced PERP transfer of waves
- Slow PAR transfer of waves

- Might also be other preferred directions!



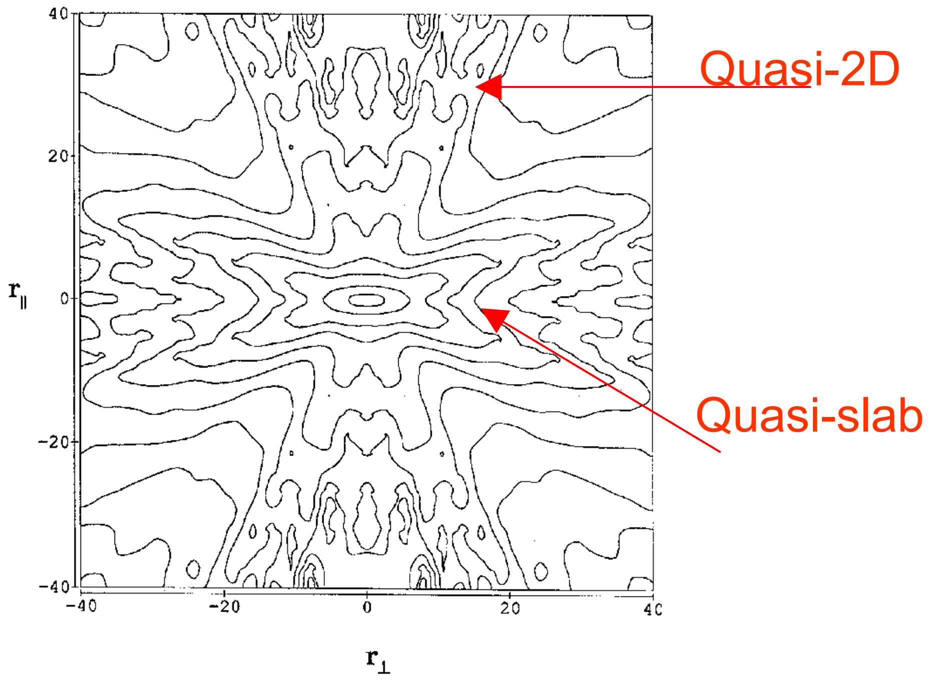


Maltese cross

- Several thousand samples of ISEE-3 data
- Make use of variability of ~1-10 hours mean magnetic field

relative to radial (flow) direction

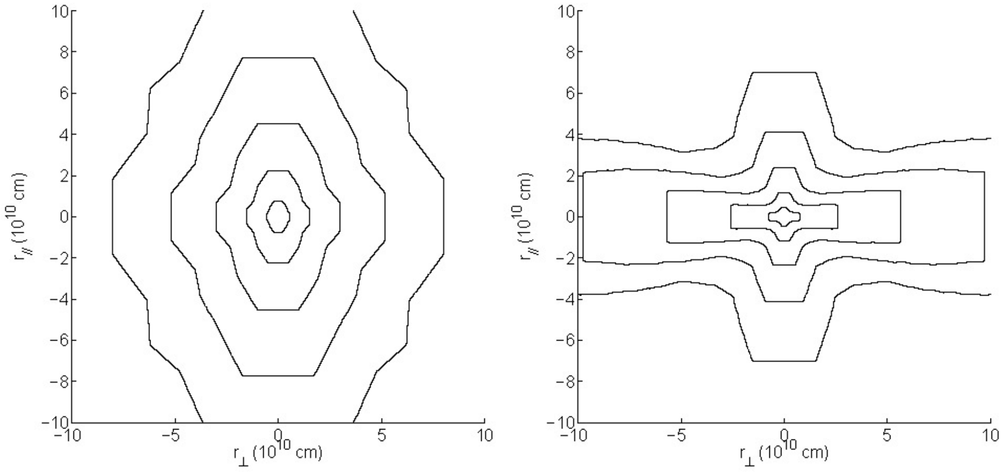
HOWEVER, FRM Dasso et al



Matthaeus+ JGR 1990

More 2D-like

More slab-like

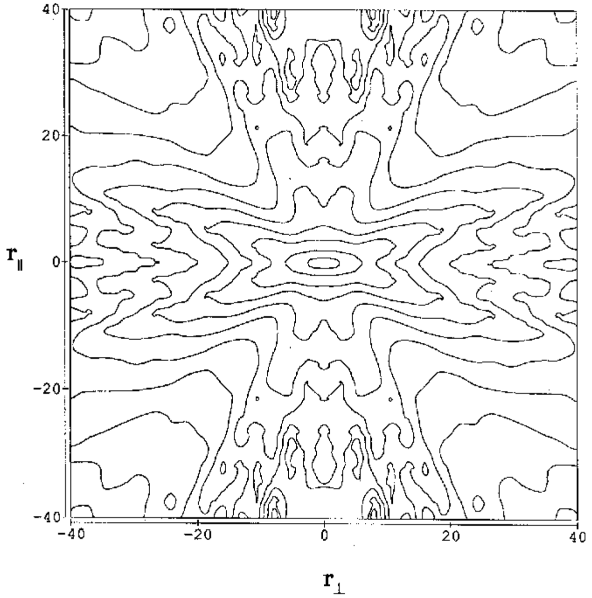


<400 km/s

> 500 km/s

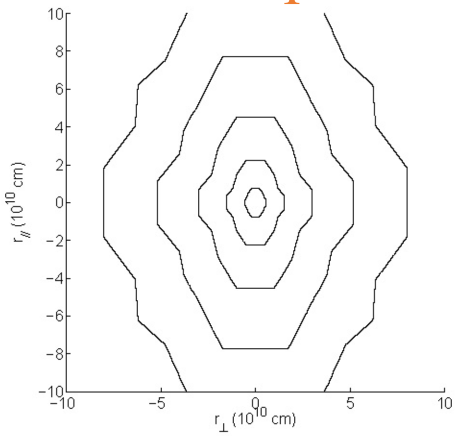
Anisotropy: What we think we know:

Maltese cross

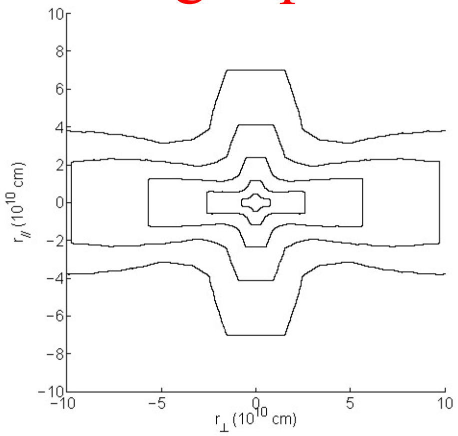


Some kind of composite of
Quasi-2D (low freq. turbulence
And quasi-slab (wavelike) fluctuations?

Low speed

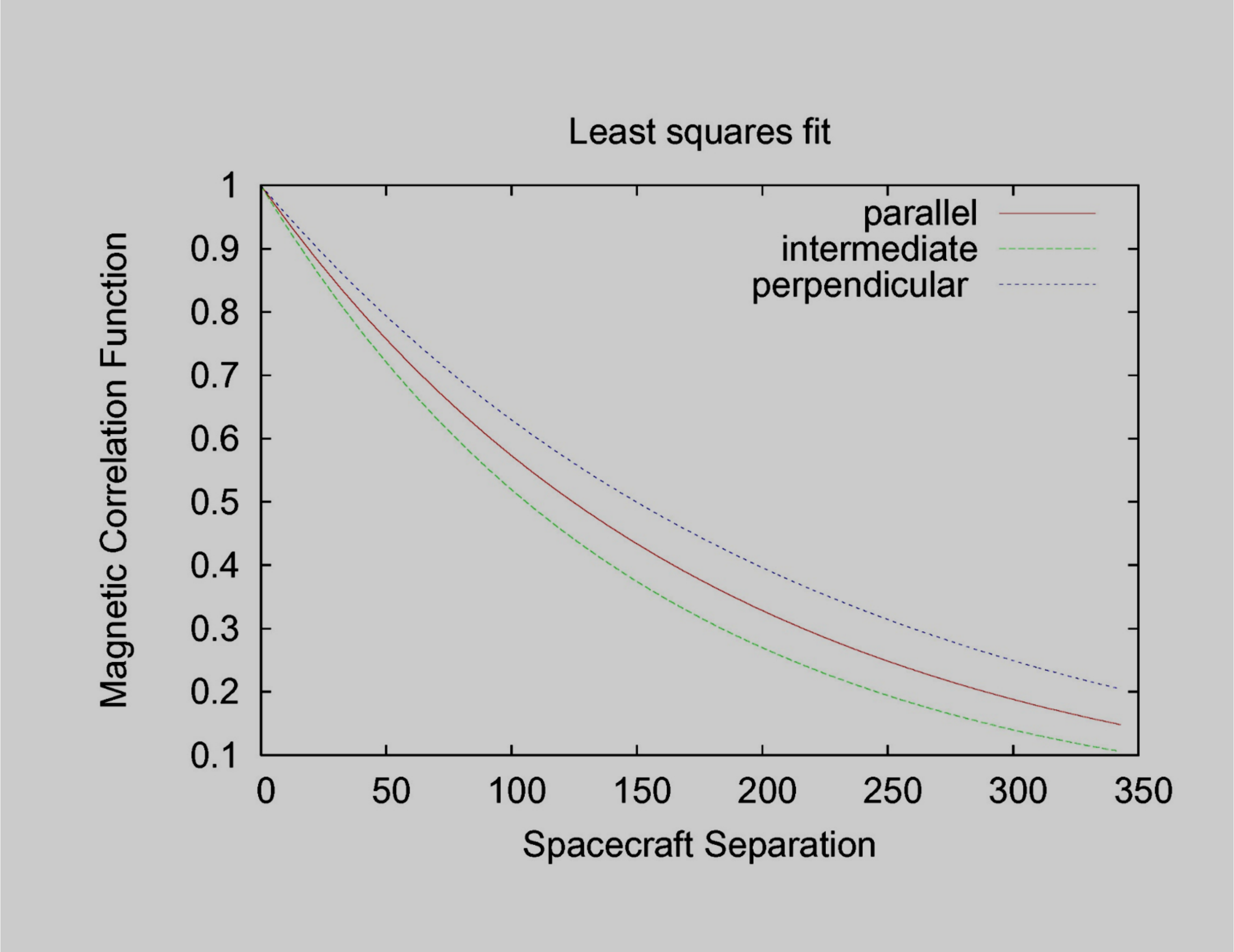


High speed



See Dasso et al,

multi s/c correlation fn. in angular channels



Anisotropy of spectra from multi s/c analysis
(ACE, Geotail, IMP8, Interball, Wind)
slow solar wind

Weygand+JGR 114, 2009

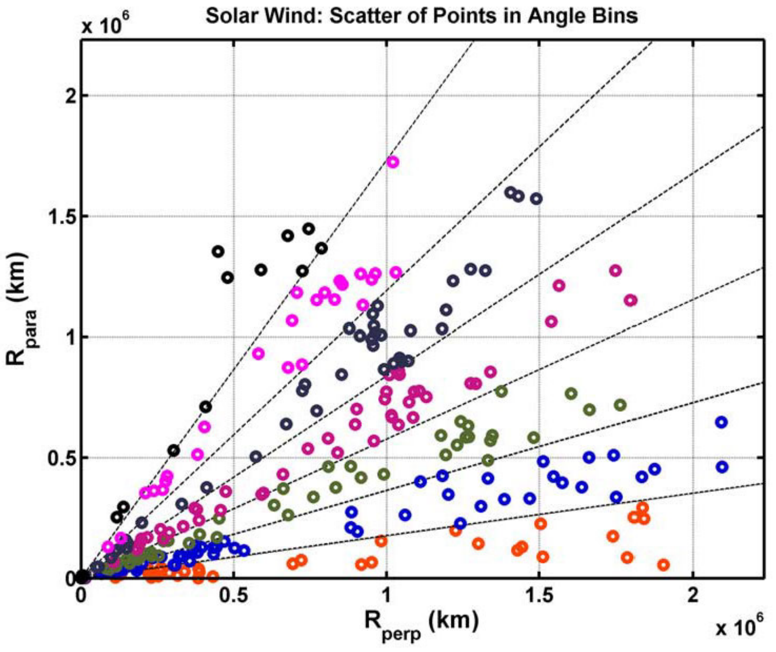


Figure 2. Distribution of spacecraft separations in the parallel and perpendicular directions with respect to the mean magnetic field direction for the slow solar wind. The dashed lines show bin boundaries, and color identifies the points within each bin.

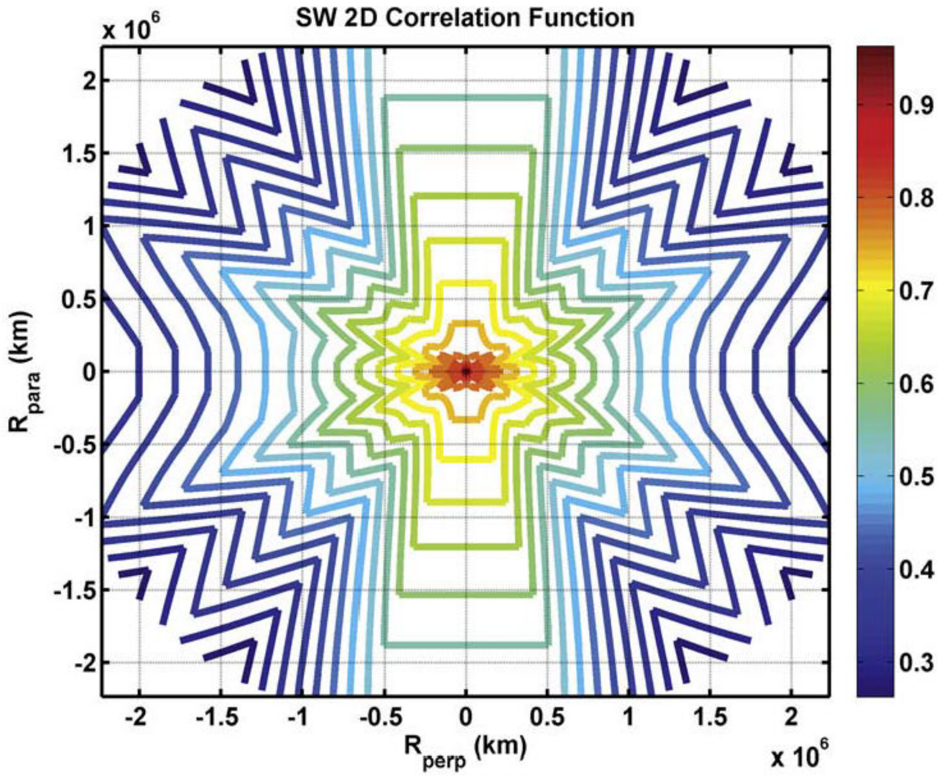


Figure 3. Correlation contour plot for the slow solar wind. The contours are calculated for one quadrant and then mirrored into the other quadrants. The color bar indicates the value of the correlation coefficient. The plot shows that the longest correlations are along the magnetic field direction and the shortest correlation are perpendicular to the magnetic field direction.

Anisotropy of correlation and Taylor scales from multispacecraft analyses

Weygand+ JGR 114, 2009

Table 2. Slow Solar Wind Values of the Correlation Scale, the Taylor Scale, and the Effective Magnetic Reynolds Number Computed From Magnetic Field Data[a]

	Correlation Scale			Taylor Scale		
Angular Range	Number of Points	λ_{CS} (10^6 km)	λ_2 (10^3 km)	Number of Points	λ_{TS} (km)	Effective Magnetic Reynolds Number $\times\ 10^6$
0° 30°	345	5.6 ± 1.6	24.1 ± 7.6	21	930 ± 160	37.0 ± 35.5
30° 40°	190	2.0 ± 0.3	14.5 ± 7.1	12	1200 ± 620	2.6. ± 3.0
40° 50°	267	2.7 ± 0.4	13.9 ± 5.1	28	700 ± 180	14.8 ± 13.2
50° 60°	345	2.0 ± 0.2	4.1 ± 2.2	52	1400 ± 230	2.0 ± 1.5
60° 70°	399	3.2 ± 0.4	12.4 ± 3.0	50	740 ± 210	18.9 ± 17.3
70° 80°	413	2.9 ± 0.4	13.8 ± 2.9	50	1000 ± 200	8.6 ± 7.1
80° 90°	439	2.1 ± 0.2	3.7 ± 1.1	48	1030 ± 290	4.4 ± 3.8

[a]Also given is the number of points that went into the determination of the correlation scale and Taylor scale values.

Quantitative measure of small scale anisotropy

- Limiting behavior of structure functions is related to "Taylor microscale"

$$S(\mathbf{l}) = R(0) - R(\mathbf{l})$$

$$\Gamma = \lim_{l \to 0} \frac{S(\mathbf{l}_\perp)}{S(\mathbf{l}_{||})} = (\lambda_{||}^T / \lambda_\perp^T)^2$$

(qualitatively similar in inertial range)

TABLE I: An estimate of $\Gamma = (\lambda_{||}^{Taylor}/\lambda_\perp^{Taylor})^2$: squared ratio of Taylor microscales.

	R_{bb}	R_{vv}	R_{out}	R_{in}	R_{vb}
Slow Wind	1.4	1.3	1.5	1.2	1.7
Fast Wind	0.6	0.5	0.6	0.4	0.7

Comparison of spectra/correlation fns from single s/c and multiple s/c analyses

- Magnetosheath and solar wind spectral estimates done three ways
- FFT spectrum 1 s/c
- Equivalent spectrum from 2nd order structure function computed 1/sc and Taylor hypothesis
- Individual points from 2s/c struciture function plotted as equivalent spectra
- Equivalent spectrum is

$$\lambda D_x^2(\lambda) \ \ vs \ \ \frac{1}{\lambda}$$

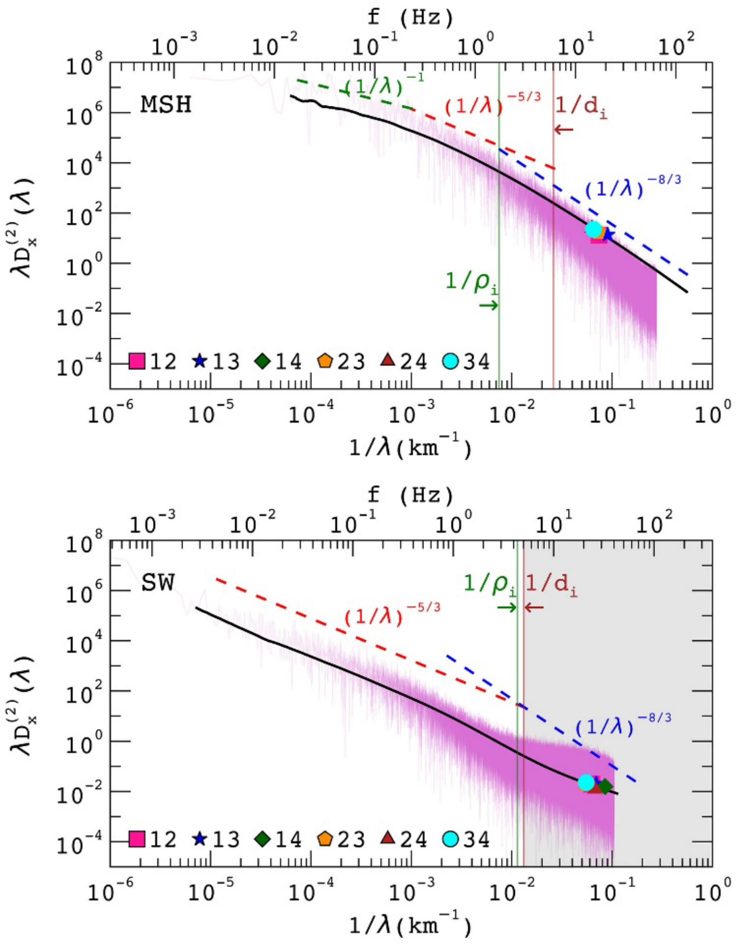


Figure 5. Equivalent spectra computed from the second-order structure function of the *x*-component of the magnetic field, in the magnetosheath (top) and the solar wind (bottom), shown as a function of equivalent wave number $k^* = 1/\lambda$, where λ is the lag (see text). The single-spacecraft MMS1 result is shown as a black solid line, and multispacecraft values are plotted as filled symbols. The fast Fourier transform spectrum of the *x*-component of **B** is shown in a translucent purple shade. Dashed lines corresponding to spectral slopes of −1, −5/3, and −8/3 are shown for reference, and the brown and green vertical lines mark the ion inertial length (d_i) and the ion gyroradius (ρ_i), respectively. The gray-shaded region in the bottom panel marks frequencies above 5 Hz, where single-spacecraft results are unreliable due to FGM instrument noise (see Appendix A). MSH = magnetosheath; SW = solar wind; MMS = Magnetospheric Multiscale.

Chhiber+JGR, 2018

IX. Estimating the space-time correlation

Space-time correlation

- Two-point, two time 2nd order correlation includes a lot of what we already discussed in separate approximations

- $R_{ij}(\boldsymbol{r}, \tau) = \langle\ b_i\,(\boldsymbol{x} + \boldsymbol{r}, t + \tau)\ b_j(\boldsymbol{x}, t)\ \rangle$

Basic ideas for analyzing space and time correlations

PRL **116,** 245101 (2016) PHYSICAL REVIEW LETTERS week ending 17 JUNE 2016

Ensemble Space-Time Correlation of Plasma Turbulence in the Solar Wind

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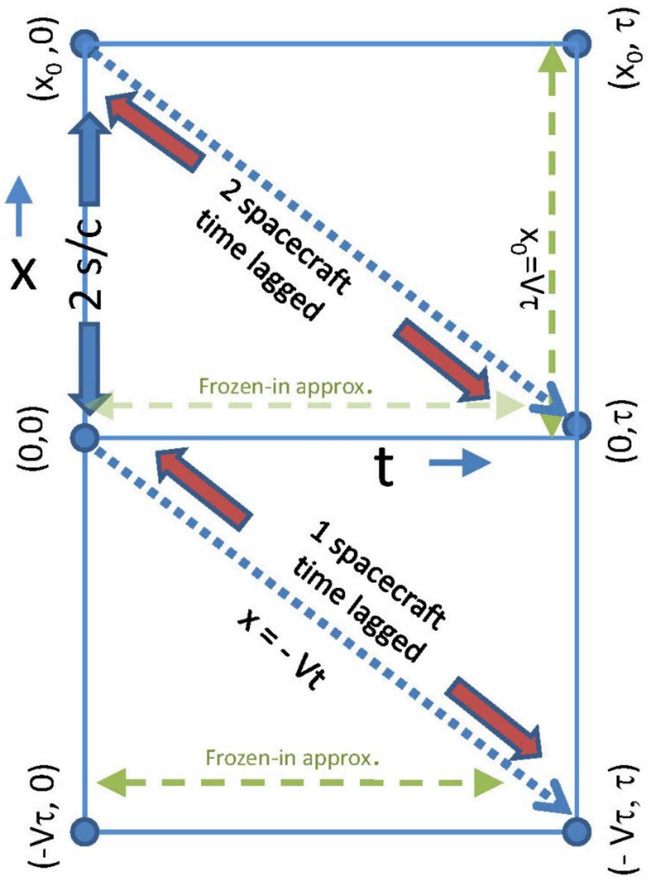
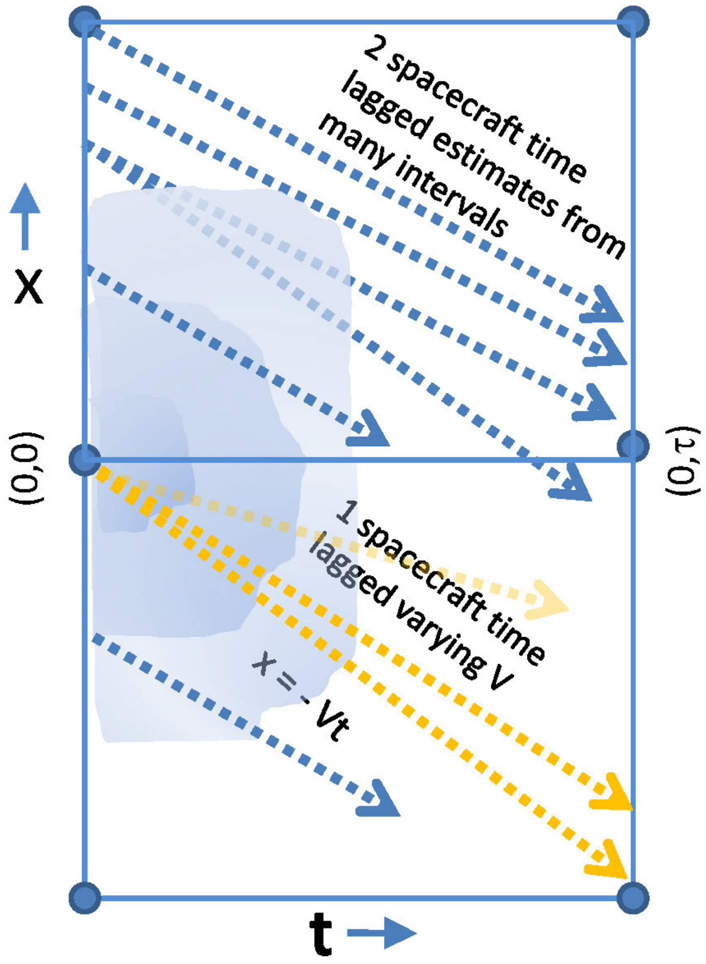
(Received 27 December 2015; revised manuscript received 12 April 2016; published 14 June 2016)

Single point measurement turbulence cannot distinguish variations in space and time. We employ an ensemble of one- and two-point measurements in the solar wind to estimate the space-time correlation function in the comoving plasma frame. The method is illustrated using near Earth spacecraft observations, employing ACE, Geotail, IMP-8, and Wind data sets. New results include an evaluation of both correlation time and correlation length from a single method, and a new assessment of the accuracy of the familiar frozen-in flow approximation. This novel view of the space-time structure of turbulence may prove essential in exploratory space missions such as Solar Probe Plus and Solar Orbiter for which the frozen-in flow hypothesis may not be a useful approximation.

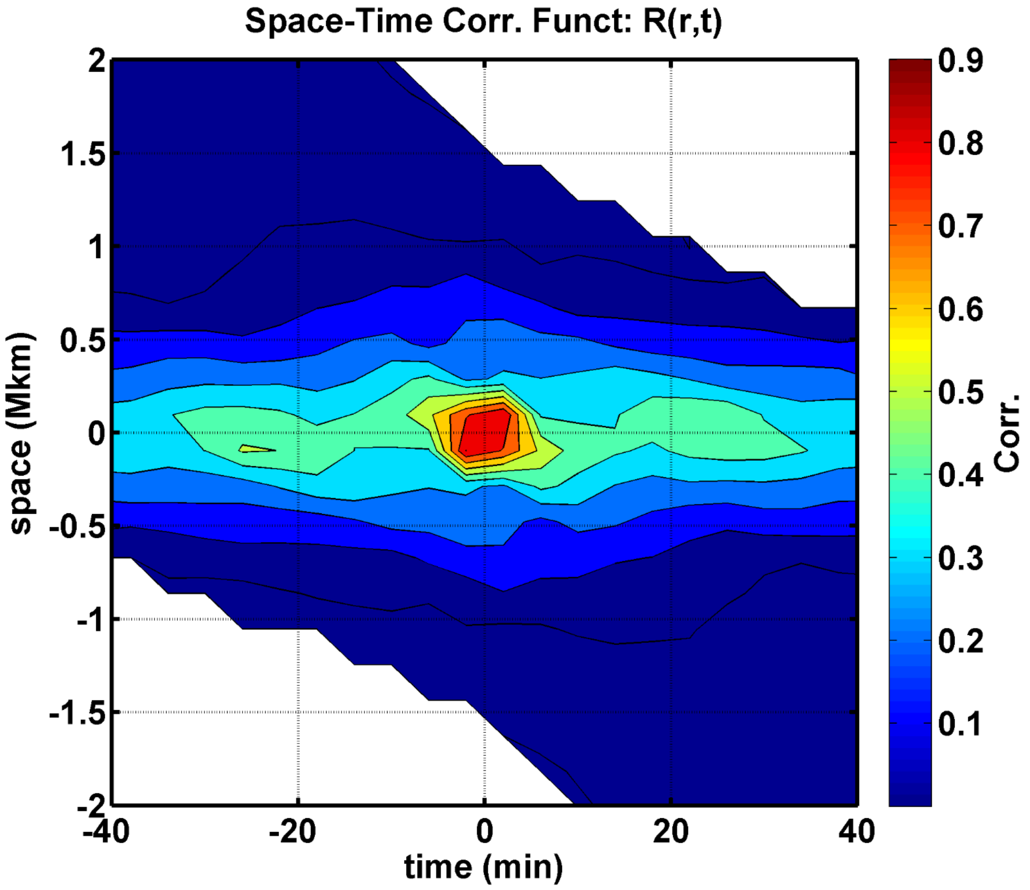
Space-time correlation

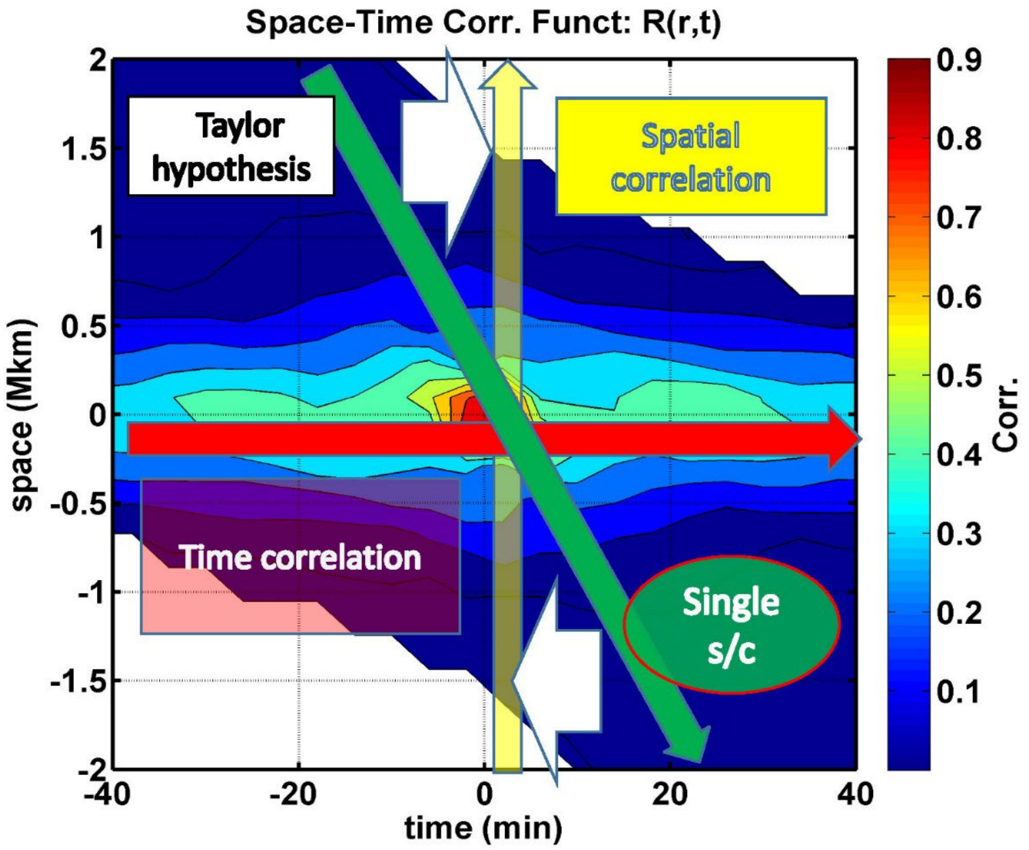
Matthaeus+ PRL 116, 245101 (2016)



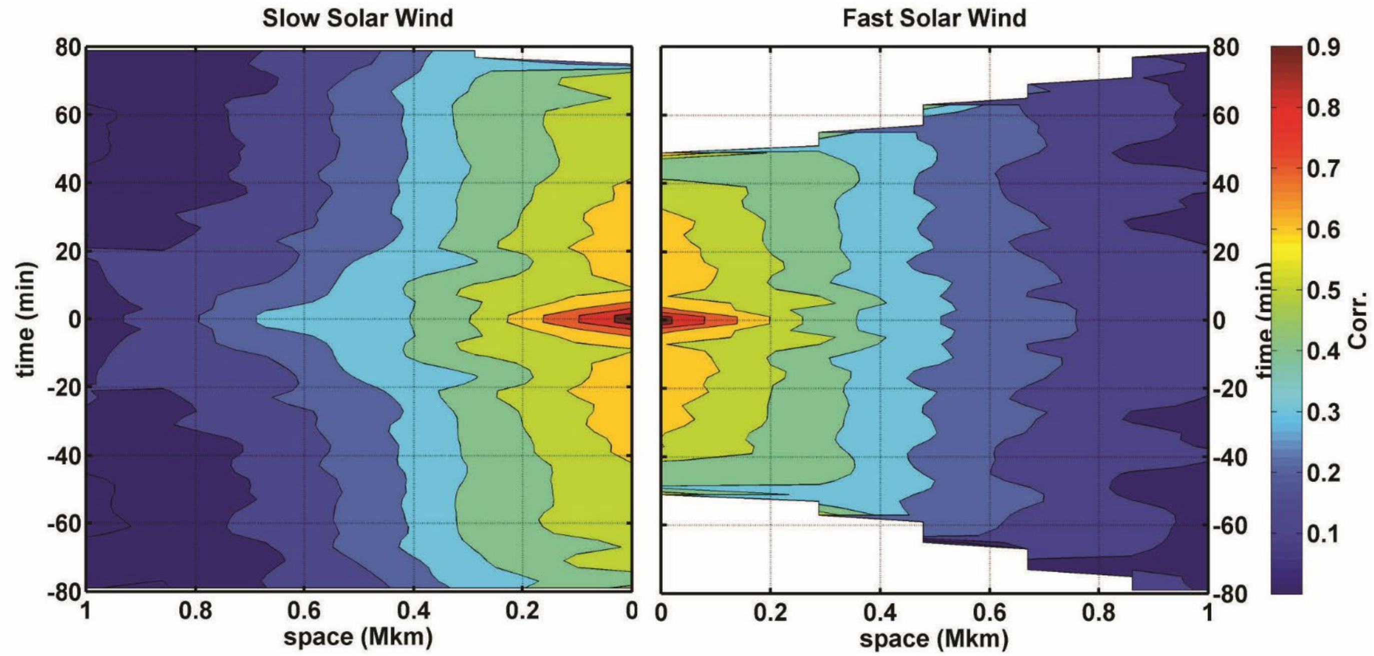


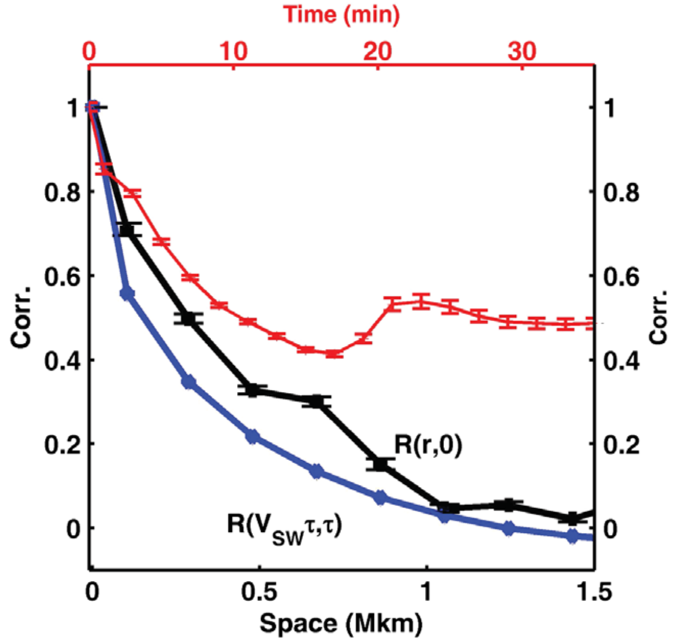
Space-time


Space-Time Corr. Funct: R(r,t)
space (Mkm)
time (min)
Corr.


Space-Time Corr. Funct: R(r,t)
Taylor hypothesis
Spatial correlation
Time correlation
Single s/c
space (Mkm)
time (min)
Corr.

Space-time correlation in sw





Pecora+PNAS 122 (2025)

MMS space time

- 6 inter s/c pairs and 1180 magnetosheath intervals to get good coverage in space and time.
- This enables estimation of the scale dependent time decorrelation (the “propagator”) a fundamental quantity in mathematical turbulence theory …see Pecora+PNAS

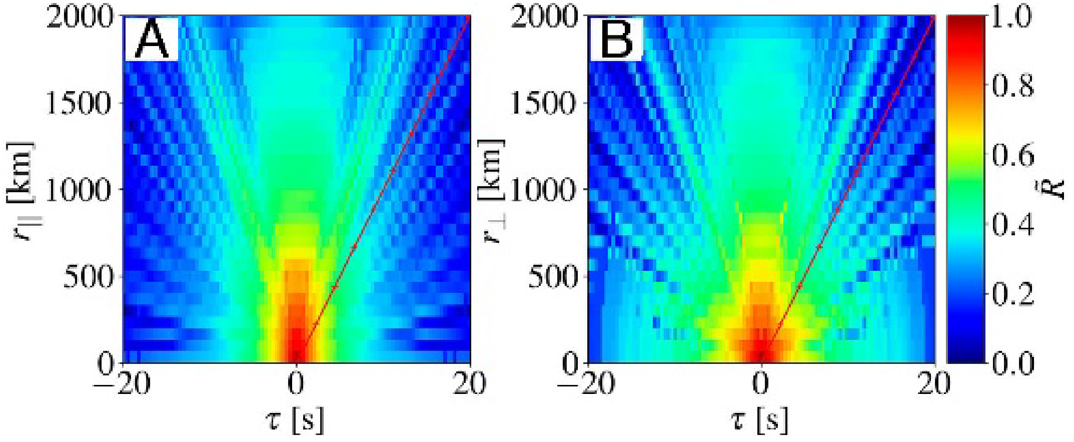
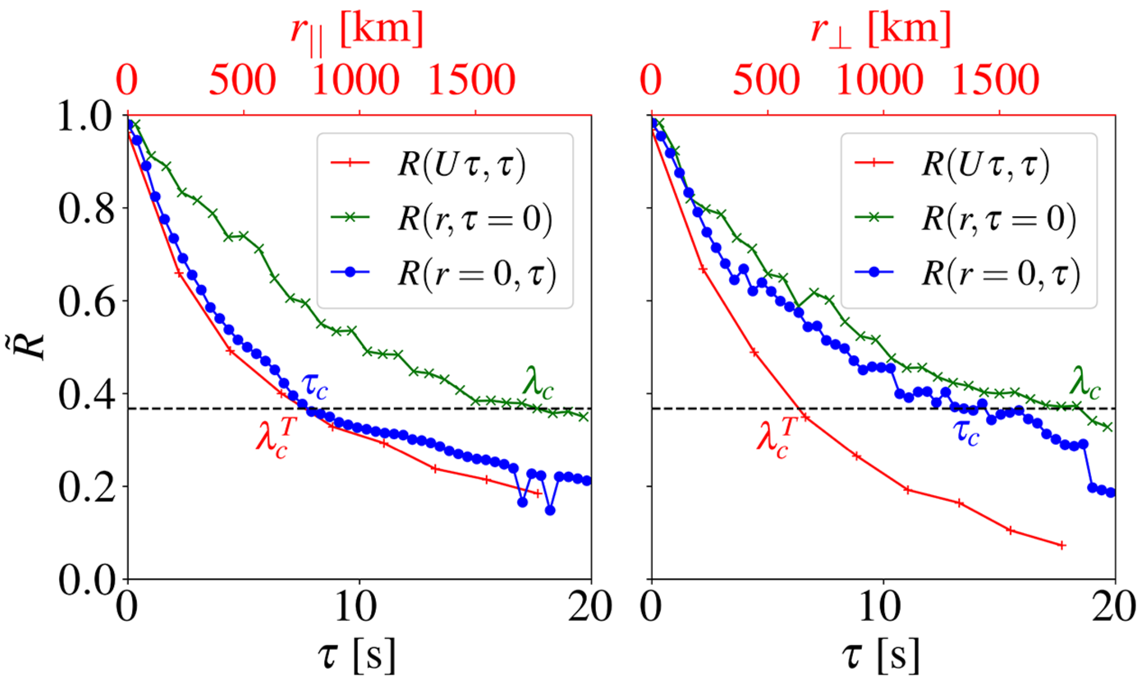


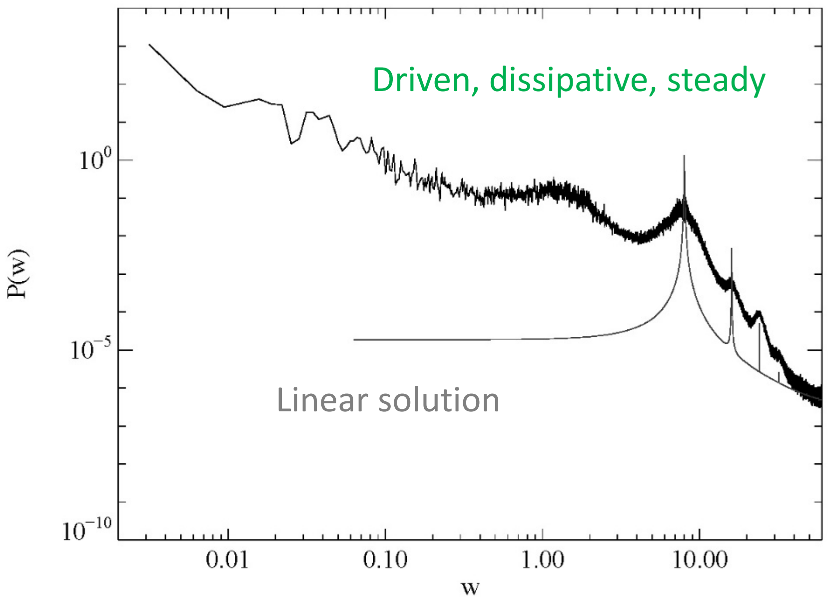
Fig. 1. Space-time correlation function with complete coverage of the $(\tau, r_{||})$ plane (*A*) and the $(\tau, r_\perp)$ plane (*B*). The red oblique line is the direction along a plasma parcel flowing at a speed of 100 km/s.

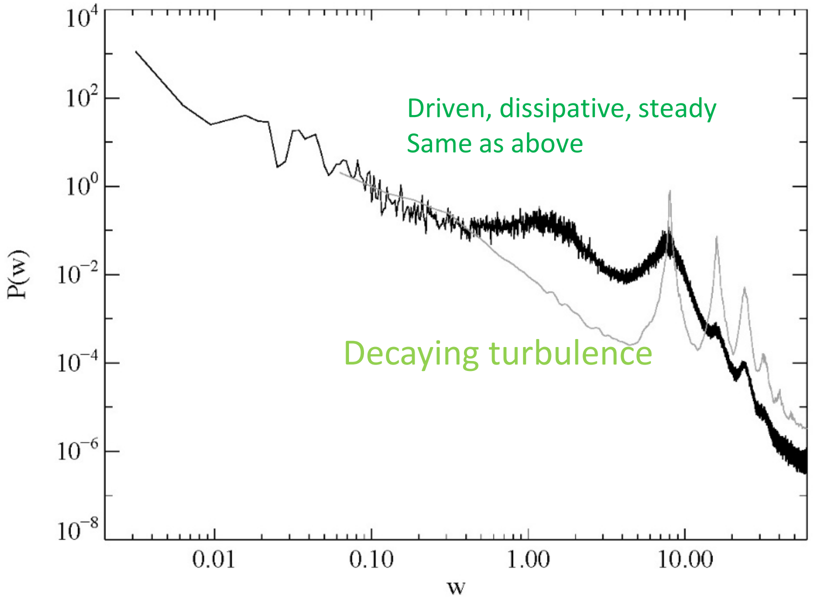


Related issue: are there any kind of recognizable "waves" in turbulence?

- Simulations of driven dissipative MHD with imposed DC magnetic field of varying strength show little indication of power in "waves" at frequencies that solve the dispersion relations
  - for *ANY* value of imposed magnetic field B0 !
- Shown are Eulerian frequency spectra (one point) with B0=8, for :
  - driven steady case
  - decaying (energy renormalized) turbulence
- Varying dB/B0 one find no more than ~16% energy in the dispersion relation peaks, With maximum at dB/B0 ~ ½
- See Dmitruk and Matthaeus, Phys Plasmas 2008

Eulerian frequency spectra





X. Intermittency and higher order statistics in multi-spacecraft measurements

Intermittency: burstiness of dissipation and of small scale gradients

- These are related by Kolmogorov Refined Similarity Hypothesis

- In general is related to structure, which can be

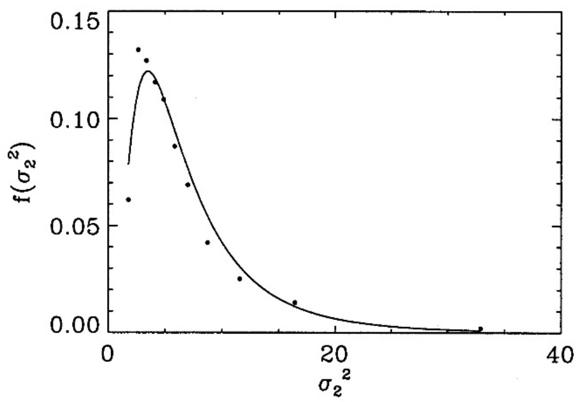
  small scale (Kolmogorov 1962)

  or large scale (Obukhov 1962)

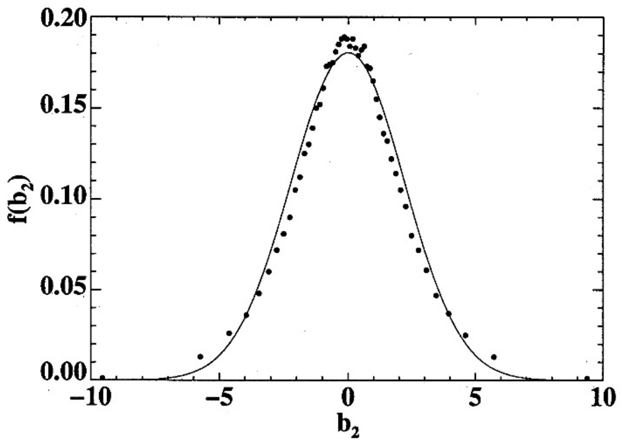
PDF of component variances

- Variances are approx. log-normal

  → Suggests independent (scale invariant) distribution of coronal sources



- When normalized to remove variability of mean and variance, component distributions are close to Gaussian



Padhye et al, JGR 2001; Sorriso-Valvo et al, 2001

- PDFs

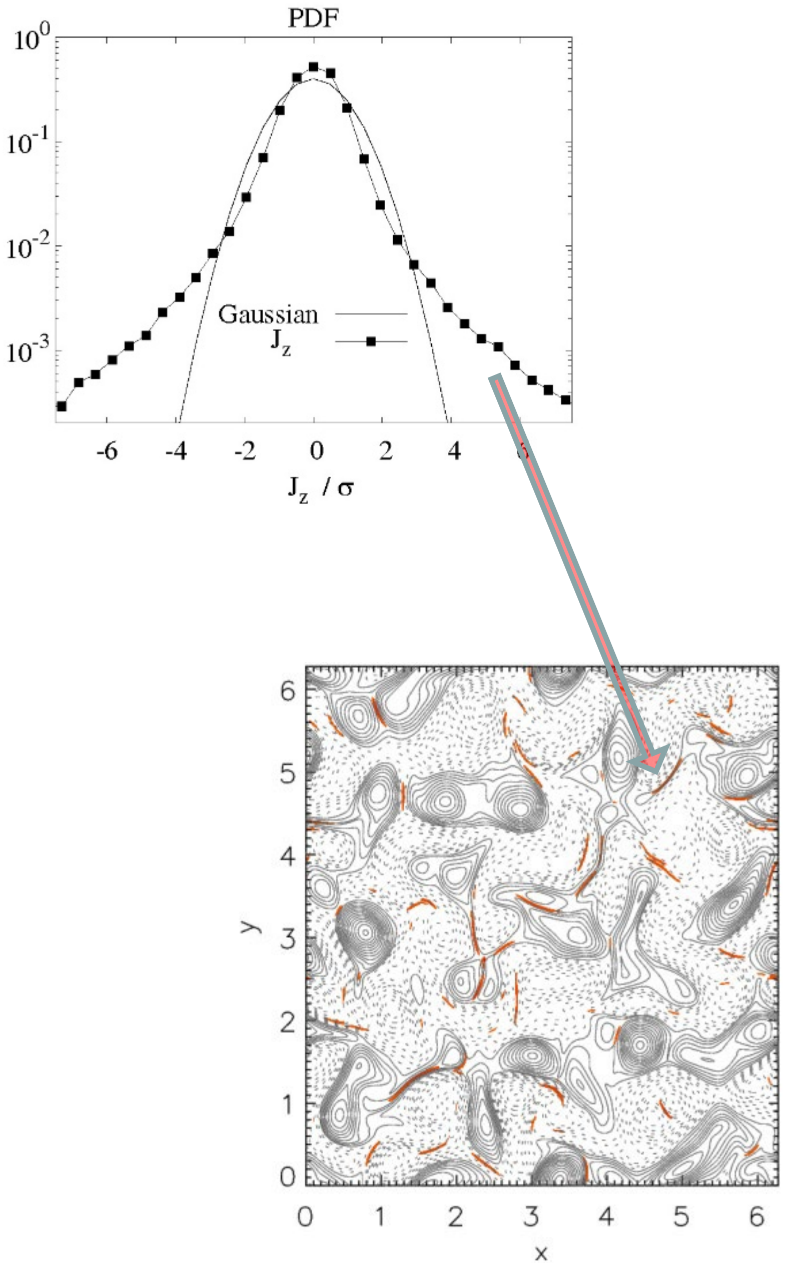
intermittency corresponds to → fat tails

for increments (or gradients)

- Kurtosis and filling fraction F

$\kappa = \langle x^4 \rangle / \langle x^2 \rangle^2$

HEURISTIC: $\kappa \sim 1/F$



Alternative views of origin of "flux tubes" and discontinuities/current sheets in SW

Spaghetti models:

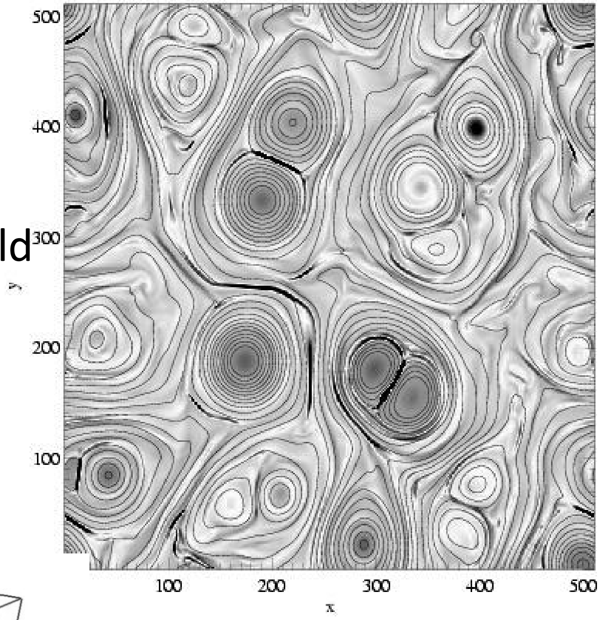
e.g.,. Bruno et al. [2004]

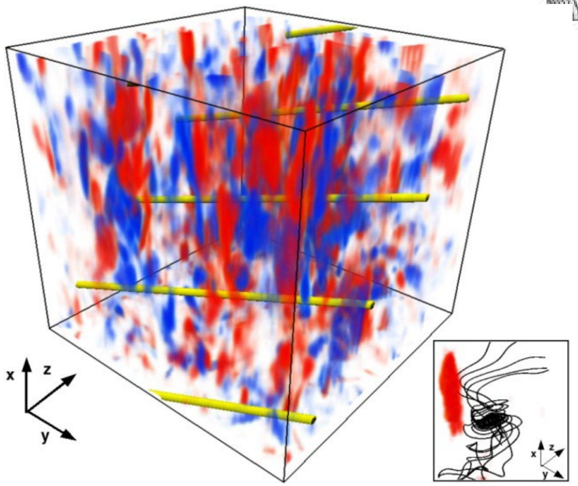
Figure 1. A sketch of the flux tube texture of the solar-wind plasma. Each flux tube contains a different plasma and the flux tubes move independently. A depiction (left) looking at the sides of the tubes indicates that the tubes are tangled about the direction of the Parker spiral. An end view (right) depicts the cross sections of the network of tubes. The scale sizes of the flux tubes correspond to the scale sizes of granules on the solar surface. The median diameter of a flux tube at 1 AU is 5.5×10^5 km.

- Passive flux tubes with boundaries -- Borovsky 2008

In turbulence: expect structure from outer scale to dissipation range

Current and Magnetic field in 2D MHD simulation



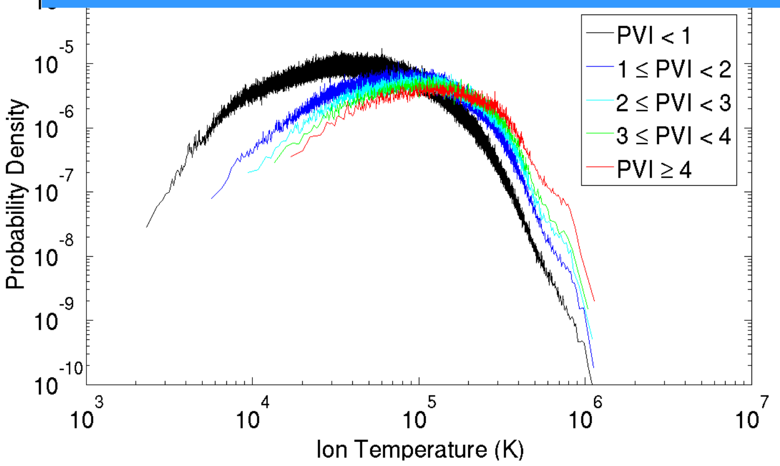


3D *MHD* compressible simulation with mean B_0

Solar wind: Sharp gradients/coherent structures

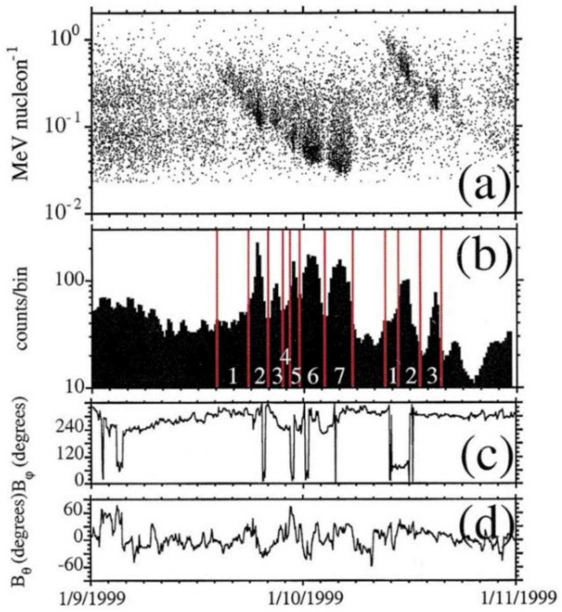
- Are hotter

Most intense current sheets associated with largest heating



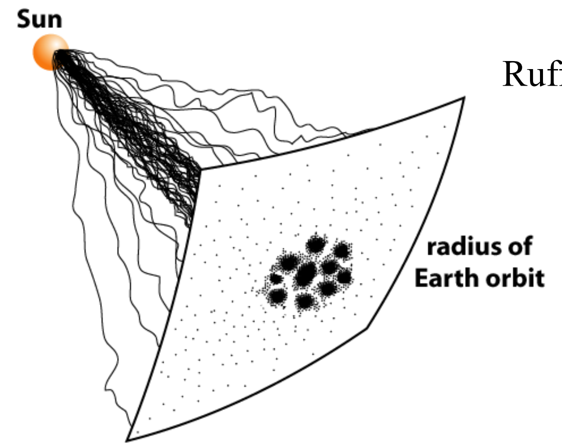
Ion temperatures, Wind s/c
Data: Osman et al, 2011

- Act as transport boundaries for suprathermal particles



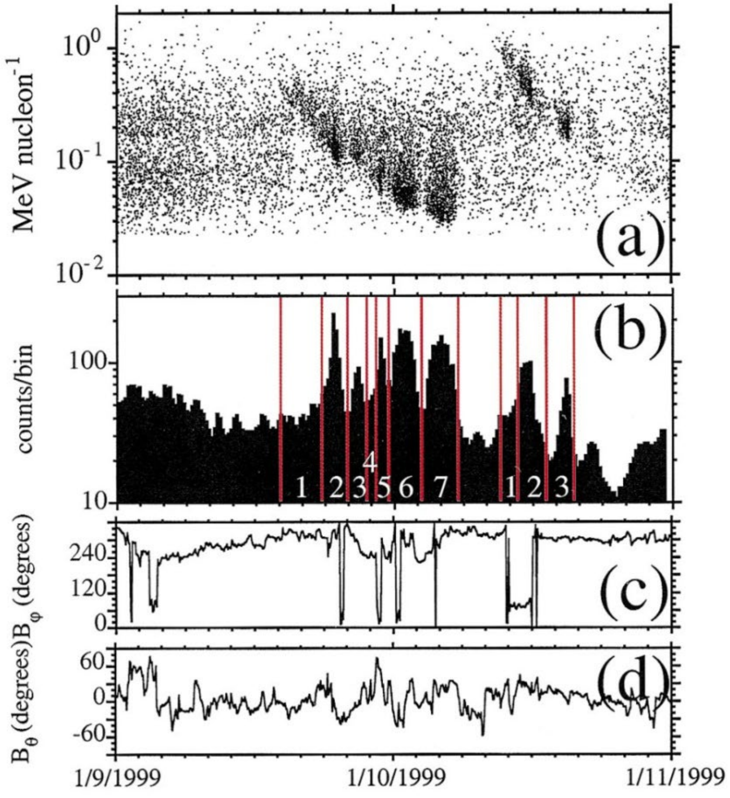
H-FE ions vs arrival time
9 Jan 1999 SEP event
[Mazur et al, ApJ (2000)]

Spatially structured turbulence is expected to have transport or "trapping" boundaries



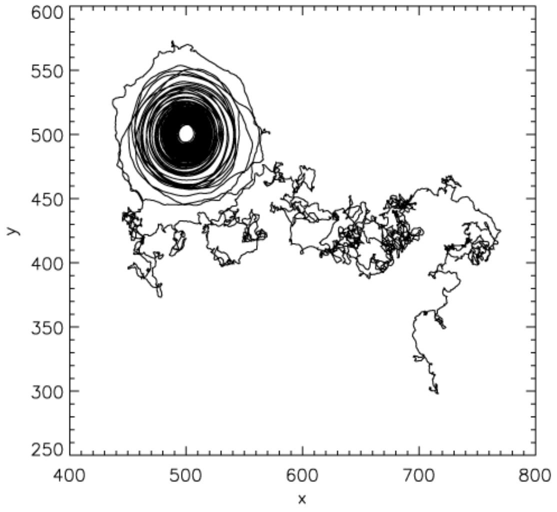
Ruffolo et al.2003

Boundaries are observed: "dropouts" of Solar energetic particles

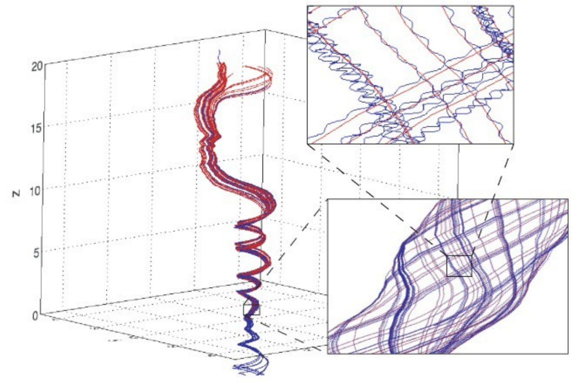


H-FE ions vs arrival time
For 9 Jan 1999 SEP event
From Mazur et al, ApJ (2000)

Model: strong flux tube plus random fluctuations



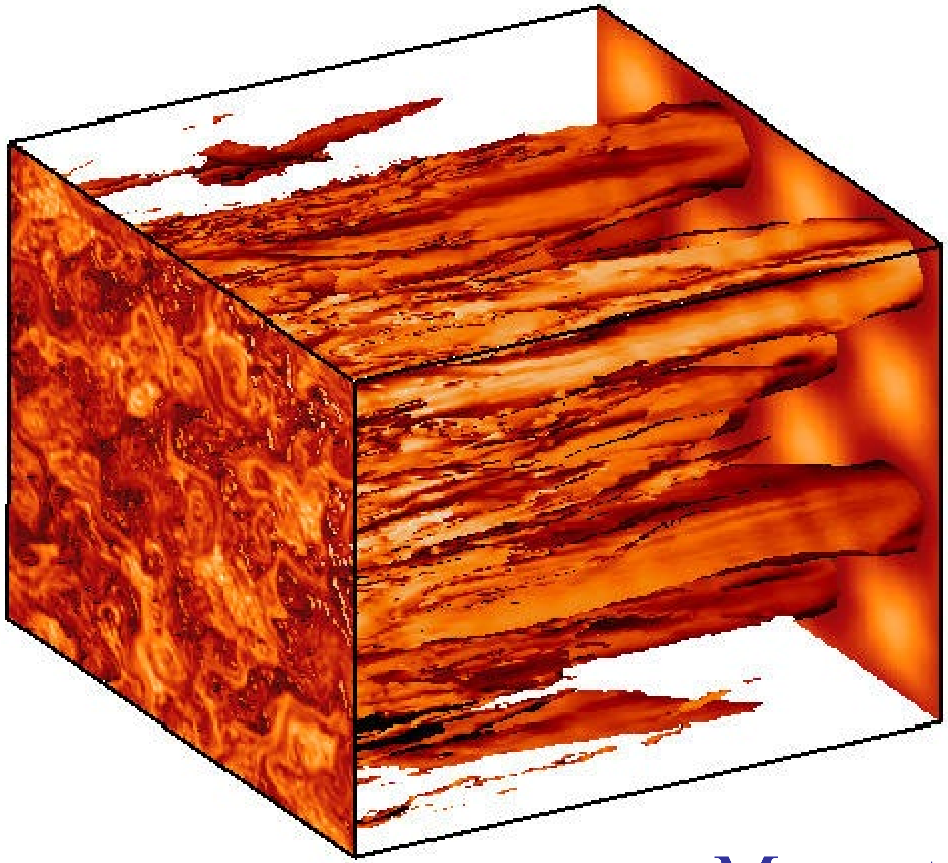
Projection of a trapped field line



Temporarily trapped Particles-- Tooprakai et al, 2007

Magnetic field lines/magnetic flux surfaces for model solar wind turbulence

A mixture of 2D and slab fluctuations in the "right" proportion



Magnetic Surfaces Composite MHD Turbulence
(80% 2D ; 20% Slab)

Magnetic structure is spatially complex

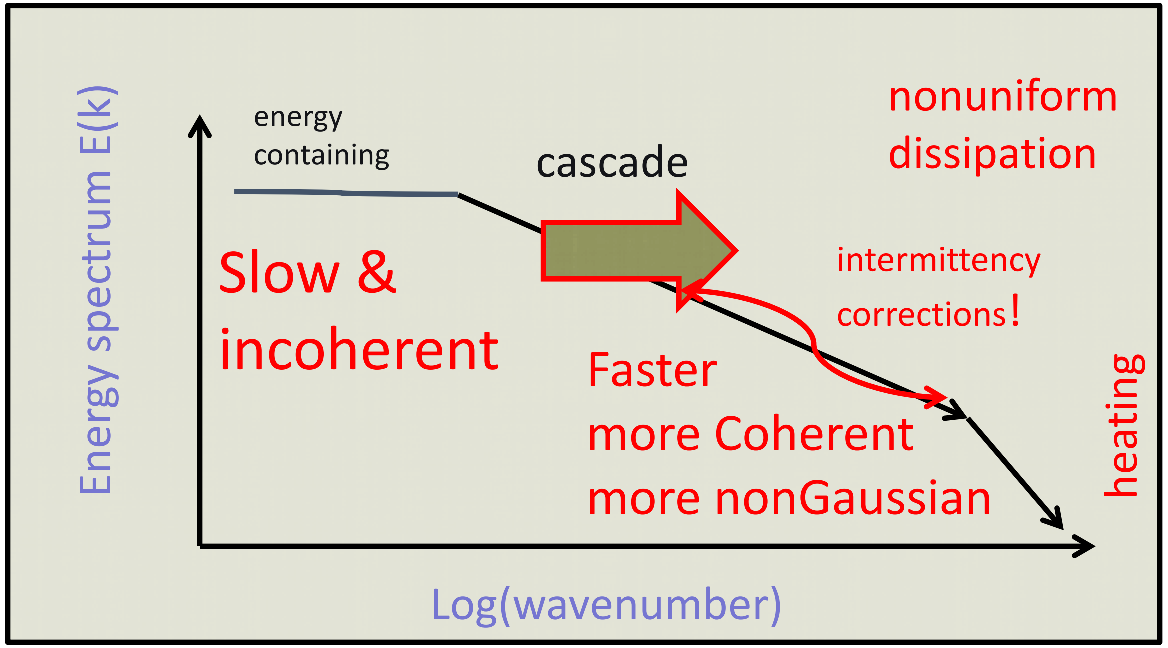
Turbulence: shear-driven kinetic plasma turbulence

Structures in electric current density

(see Karimabadi et al, PoP 2013)

Thinnest sheets comparable to electron inertial length. Sheets are clustered at scales near ion inertial length

More detailed cascade picture



- **Cascade:** *progressively enhances nonGaussian character*
- *Generation of **coherent structures** and patchy correlations*
- *Coherent structures are sites of **enhanced dissipation***

Coherent Structure Detection

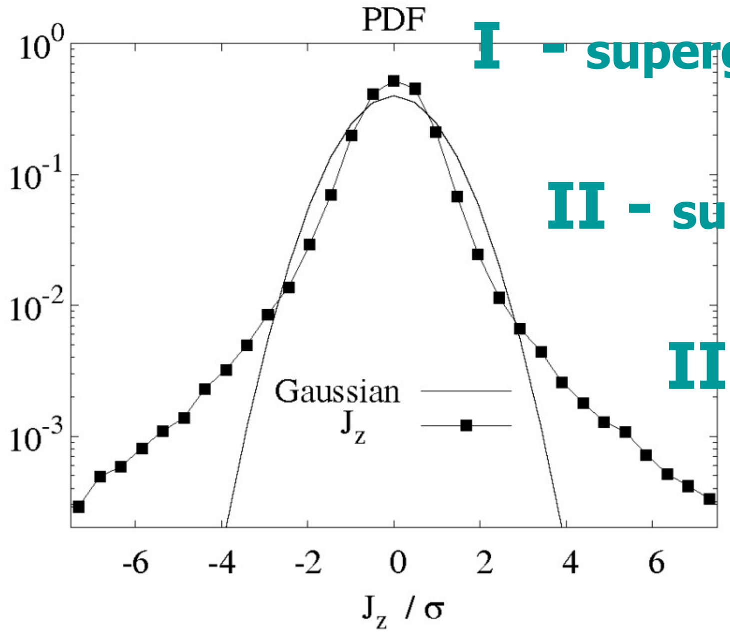
- Normalized partial variance of increments (PVI).
  - Related to turbulence intermittency.

$$PVI = \frac{|\Delta \mathbf{B}|}{\sqrt{<|\Delta \mathbf{B}(\mathbf{x})|^2>}}$$

  - Where the vector increment is $\Delta\mathbf{B}(x) = \mathbf{B}(x + s) - \boldsymbol{B}(x)$
- The PVI statistic is constructed such that:
  - $< PVI^2 > = 1$
  - $< PVI^4 >$ is related to kurtosis.
  - Higher powers are connected with familiar intermittency diagnostics (multifractals, etc).
- Use PVI statistics to determine if spatial patches are bounded by current sheets.

Intermittency and the spatial organization of current density

N.B. “PVI” acts a lot like the current



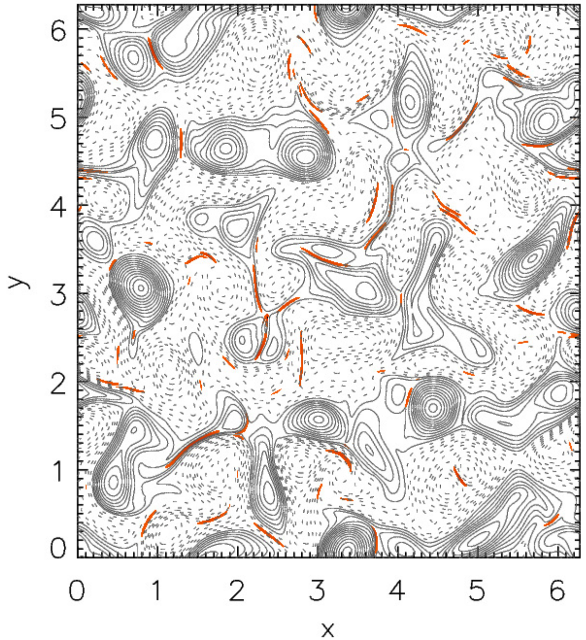
I - supergaussian

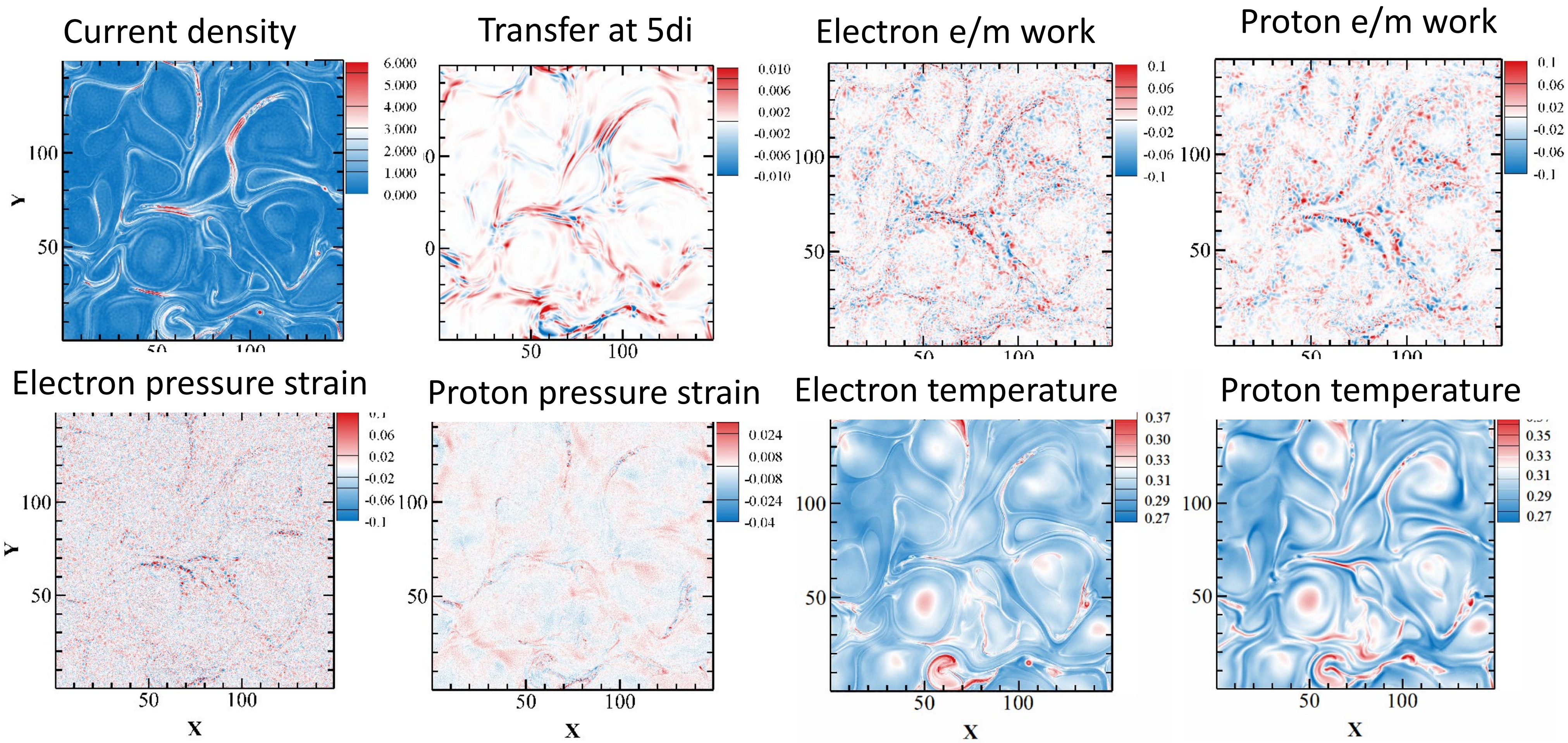
II - subgaussian

III – supergaussian-
(PVI> 4)

Greco et al, 2009

III – supergaussian
current sheets

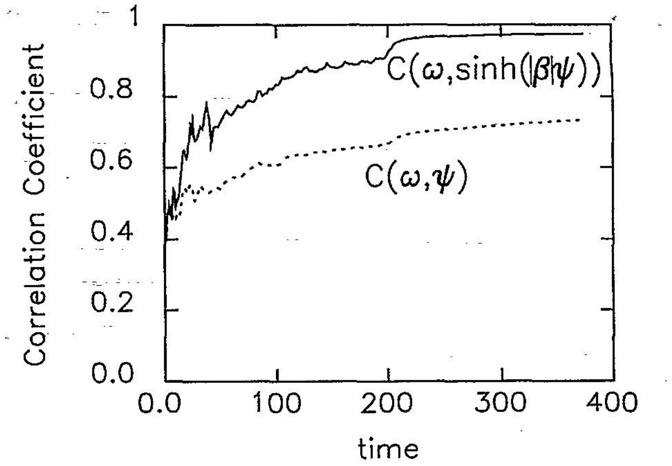




Turbulence in kinetic plasma – "intermittency" and structure in *every relevant quantity*!

2D NS max entropy

System approaches Max Ent state C > 0.97 →

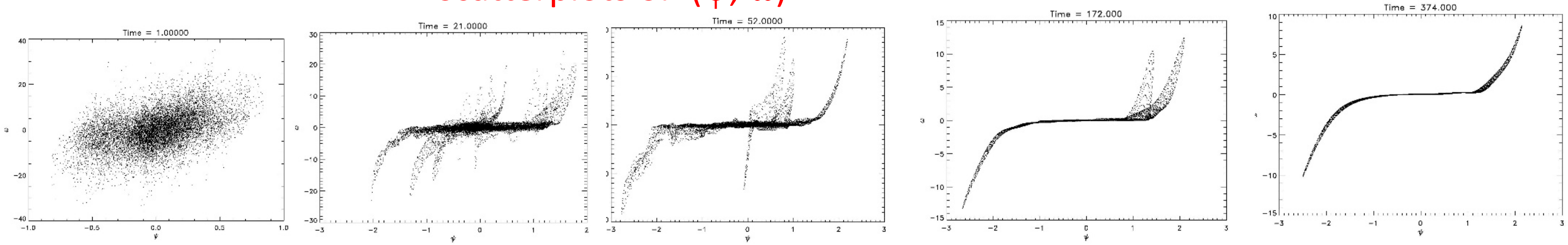


Vortices isolate, axisymmetrize, locally relax, collide, merge, etc

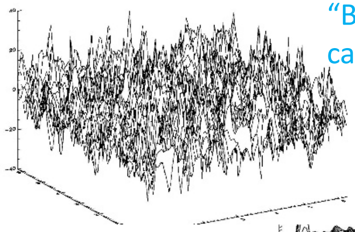
If equilibrium →

$\boldsymbol{\omega = \omega(\psi)}$

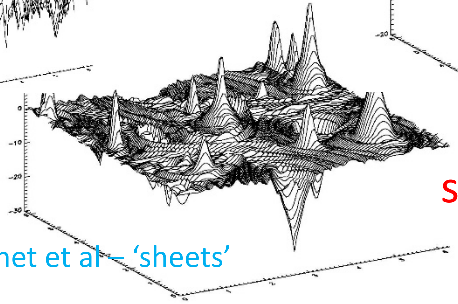
scatterplots of (ψ, ω)



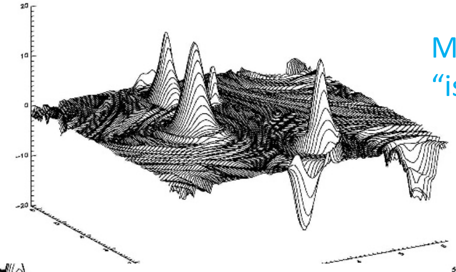
time = 1 t= 21 t=52 t=172 t=374



"Batchelor" cascade

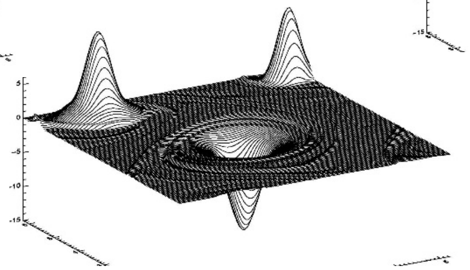


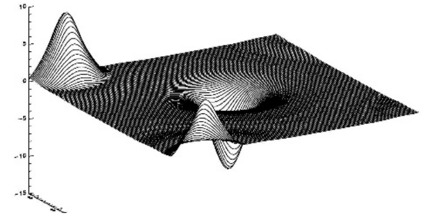
Brachet et al – 'sheets'



MacWilliams – "isolated'

surfaces of ω(x,y)





Montgomery-Joyce - `Max entropy'

Montgomery et al, 1992

Higher order statistics

- Correlations using primitive variables such as <bi bi bj bj’>
- Structure functions such as S4 =<(delta bi)^4> with increments delta bi = bi-bi’
- Intermittency :

  nonGaussian distributions

  higher than second order statistics

  multifractal scalings --

- Third order laws: Cascade rates , involving S3= =<(delta bi)^3>

Comparison of several orders of structure functions from 1 s/c and multiple s/c analyses

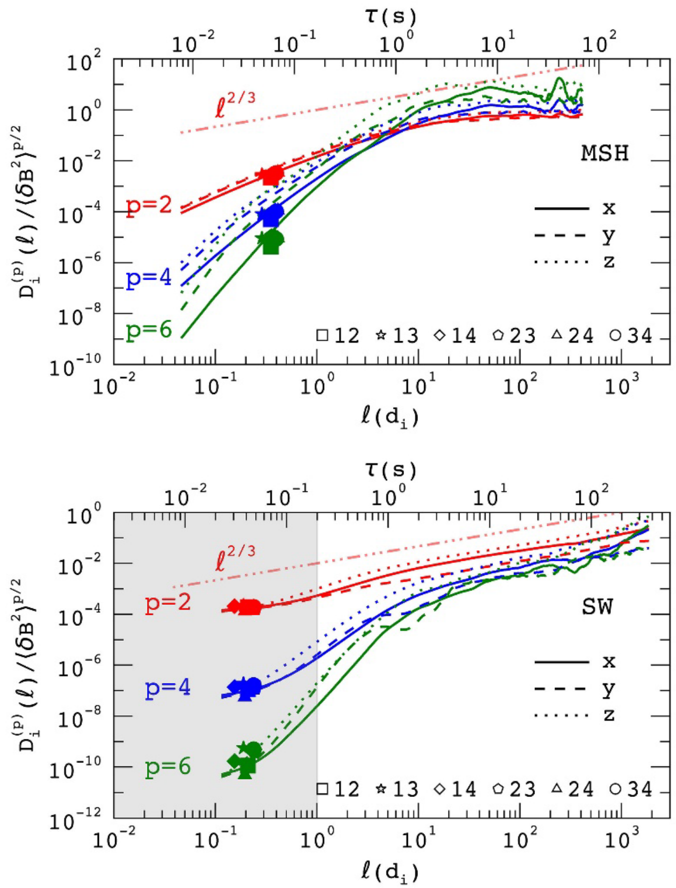


Figure 3. Second, fourth, and sixth-order structure functions of B_x (solid lines) in magnetosheath (top) and solar wind (bottom). Structure functions of B_y and B_z are shown as dashed and dotted lines, respectively. Two-spacecraft structure functions of B_x are plotted using filled symbols that indicate different spacecraft pairings, labeled in the bottom right corner. Structure functions of order p are normalized by the pth power of the root-mean-square magnetic fluctuation, and the lag ℓ is expressed in units of the ion inertial length d_i. Single-spacecraft structure functions have been computed using MMS1 measurements. The gray-shaded region in the bottom panel marks frequencies above 5 Hz, where single-spacecraft results are unreliable due to flux-gate magnetometer instrument noise (see Appendix A). MMS = Magnetospheric Multiscale; MSH = magnetosheath; SW = solar wind.

Chhiber+JGR, 2018

XI. Summary and topics not covered

Summary

- describing turbulence requires multipoint measurements
- to reveal three dimensional properties
- to separate spatial and temporal properties, the distinction being fundamental in turbulence
- Basic studies have looked at 2nd order spatial, second order time correlations, and some higher order statistics, and anisotropy
- Correct determinations of correlation length and Eulerian decorrelation tome are possible with these methods.
- The space-time correlation includes among other things the accuracy of the Taylor hypothesis
- This is a good start…but there's a lot more…

Topics not covered/incompletely covered

- Measurement of cascade rates from “third order laws”
    - Yaglom/Politano Pouquet
    - Direction averaging (Nie Tanveer, Verdini+, Wang +; other recent work
    - LPDE (see Servidio, these lectures)
    - Comparison of different estimations methods (Bandyopadhyay these lectures;
      and new work, e.g., Gao+ arXiv:2605.03271v1 (2026)
- Dissipation/conversion into internal energy – pressure strain (Yang+; Riddhi+; Burch+; Bandyopadhyay these lectures)
- Refined evaluation of scales (Bandyopadhyay, these lectures)
    - Taylor microscales
    - Effective Reynolds numbers
- Magnetic field curvature (Yang+, Bandyopadhyay+, Lemoine)
- Much more in the literature….this has just been a primer

end